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HORIZONTAL VLF TRANSMITTING ANTENNAS NEAR THE EARTH

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FOREWORD

The work described in this report was accomplished in the Electronics Division, Research Department, Naval Ordnance Laboratory, Corona, California, as part of the NOLC Very Low Frequency Research Program. This program is sponsored by the Defense Communications Agency, Code 333, under MIPR's 43-4-105 and 43-5-59.

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ABSTRACT

Problems encountered in VLF radiation are outlined and variations of the horizontal dipole are proposed as a solution. Equations are derived for the radiation patterns, input impedance, and efficiency of the broad-band antenna (terminated in its characteristic impedance) and of the narrow-band antenna (open-terminated) where the antenna is fed at any position along its length. Data are presented to substantiate that (1) the horizontal dipole near the earth exhibits super gain when several parallel conductors are used; (2) the radiation efficiency of "n" closely spaced conductors is "n" times greater than when the same conductors are spaced one-half wavelength apart; and (3) the lossy horizontal dipole normally does not resonate when its length is a multiple of one-half wavelength. The "lossy lengthening factor" is shown in a plot of the resonant length as a function of "Q." This plot applies generally to all lossy dipoles.

CONTENTS

SYMBOLS

a	Antenna conductor radius
c	Free-space wave velocity (3×10^8)
C_p	Antenna distributed capacitance to ground (farads/meter)
d	Differential
E_f	Electric field strength (volts/meter)
E_h	Far electric field
E_θ	Electric field strength through the antenna axis in the azimuthal plane
E_ϕ	Electric field strength through the antenna axis in the elevation plane
f	Frequency
h	Antenna height (meters)
I	Antenna rms input current
I_{in}	Antenna input current
I_m	Imaginary term
I_t	Total antenna rms input current
K	A propagation constant in free space
L	Antenna series inductance/unit length
ℓ	Antenna length
ℓ_n	Distance from current maximum to end of conductor
ℓ_o	Antenna resonant length

ℓ_λ	Antenna length (free-space wavelengths)
P_{in}	Antenna input power
P_r	Radiated power
Q	$\frac{\omega L}{r}$
r	Antenna series resistance/unit length
R	Antenna range (meters)
R_{ac}	Conductor ac resistance
R_e	Real term
R_{in}	Antenna input resistance
R_m	Mutual resistance
R_{oin}	Antenna resonant input impedance
R_r	Antenna radiation resistance
R_s	Antenna self resistance
s	Distance between conductors
v	Wave velocity along antenna
V_r	Voltage at open end of antenna
x	Distance from antenna feed point
X_c	Antenna capacitive reactance
X_L	Antenna inductive reactance
X_m	Antenna mutual reactance
X_s	Antenna self reactance
Z_{in}	Antenna input impedance
Z_o	Antenna characteristic impedance
Z_t	Termination impedance

α	Antenna attenuation constant
β	$\frac{2\pi}{\lambda}$
β_1	$\frac{2\pi}{\lambda}\left(\frac{c}{v}\right)$; propagation phase constant along antenna
δ	Skin depth (meters)
ϵ_o	Free-space dielectric constant (8.85×10^{-12})
η	Antenna radiation efficiency compared to that of a perfect monopole
θ	Angle in azimuthal plane measured from antenna axis
λ	Free-space wavelength
μ	$4\pi \times 10^{-7}$, permeability of earth
σ	Earth conductivity (mhos/meter)
ϕ	Angle in elevation plane measured from antenna axis
Φ	Combined elevation and azimuthal angle measured from antenna axis
ω	$2\pi f$ (angular velocity)

INTRODUCTION

In recent years there has been a resurgence of interest in very low frequency (VLF) radio transmissions. VLF applications that once were regional are now global; it is used for communication, navigation, and standard time keeping. Many more uses would appear were it not for the great difficulty in radiating VLF. The most modern VLF transmitting antenna, which has just been completed at North Cape, Australia, cost more than $25 million. This antenna is a vertical monopole with a very narrow bandwidth (50 Hz) which is voltage-limited. It can radiate only 70 kW with 4 percent efficiency at 10 kHz. This antenna must be constructed over a large and costly high conductivity ground plane in contrast to the horizontal dipole which needs no ground plane and, in fact, operates more efficiently over low conductivity earth. The cost of the horizontal antenna is about 1 percent that of an equally efficient vertical antenna. The reason this antenna has not been used is that its characteristics are not well understood.

The purpose of this report is to present equations and curves showing the characteristics of the horizontal antenna so that its performance can be compared to that of other types of transmitting antennas.

VLF RADIATION PROBLEMS

The most difficult problem in radiating VLF energy is the design of a low cost, easily constructed transmitting antenna that will radiate efficiently. Another problem is the limitation on the amount of power that can be radiated. The largest VLF transmitting antennas, located at Cutler, Maine, and Northwest Cape, Australia, can radiate only 70 kW at 10 kHz. The narrow bandwidth of existing VLF vertical radiators is also a problem in some VLF systems. A problem of lesser concern is the difficulty of launching VLF waves at a low elevation angle.

All of these problems can be solved by the use of parallel conductors laid over low conductivity earth. The conductors used should be about 1 or 2 wavelengths long and spaced closely enough (0.03 λ) to be tightly coupled in the radiation field but isolated from each other's induction fields. When so arrayed, "n" closely spaced conductors will radiate "n" times as much power as when they are spaced $\lambda/2$ apart. The efficiency of this type of antenna increases linearly with the number of parallel

conductors used. The greatest problem encountered is that of locating a large expanse of low conductivity earth.

The horizontal dipole antenna has the capability of radiating much higher power than the vertical monopole now in use. At 10 kHz this type of antenna will radiate 100 times as much power as the largest VLF transmitting antenna in existence today. There is no bandwidth problem with the horizontal antenna; frequencies over a bandwidth of 20 to 1 can be transmitted simultaneously when this antenna is terminated in its characteristic impedance. Because of the finite conductivity of the earth, this antenna also has a low elevation launch angle.

VLF RADIATION PATTERNS

The radiation patterns of the horizontal conductor near the earth are dependent upon the feed point position along the conductor and the method of termination at the conductor ends. Once these are selected, equations for the azimuthal pattern can be written in terms of the attenuation factor, $\alpha\lambda$, the wave velocity factor, c/v, and the antenna length in wavelengths, ℓ_λ. The factors $\alpha\lambda$ and c/v are nearly constant over the VLF band, while ℓ_λ naturally varies with frequency. The shape of the elevation pattern is a function of all the variables mentioned above and also is changed somewhat at low elevation angles by the earth's reflection coefficient, which is a function of both frequency and the earth's conductivity. The planes of both the azimuthal and elevation patterns are through the axis of the conductor.

The radiated fields of the conductor near the earth can easily be derived from the field radiated by a horizontal conductor in free space. If the conductor has uniform current along its incremental length, the far electric field due to this incremental length is[1]

$$d\,E_h = \frac{60\pi(I\,d\ell)}{R\lambda}\sin\phi \tag{1}$$

The conductor is now brought down near the earth; therefore, the radiated field must be modified by the earth's reflection coefficient.[2] In the elevation plane, through the axis of the conductor, the differential radiated electric field is

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[1]Krauss, J. D., "Antennas," McGraw-Hill Book Co., New York, N. Y., 1950, first edition, p. 135.

[2]Golden, R. M., R. S. Macmillan, and W. V. T. Rusch, "A VLF Antenna For Generating a Horizontally Polarized Radiation Field," Technical Report No. 2, Calif. Inst. of Tech., July 1957.

$$dE_{\phi} = j\frac{60\pi}{R\lambda}(I\,d\ell)\left(\frac{2\sqrt{\frac{\omega\epsilon_o}{j\sigma}}}{\sqrt{\frac{\omega\epsilon_o}{j\sigma}} + \sin\phi}\right)\sin\phi \qquad (2)$$

and in the azimuthal plane the differential radiated electric field is

$$dE_{\theta} = j\frac{60\pi}{R\lambda}(I\,d\ell)\left(2\sqrt{\frac{\omega\epsilon_o}{j\sigma}}\right)\cos\theta \qquad (3)$$

The propagation constants along the conductor near the earth must be considered for an antenna of finite length. The relationship of the amplitude and phase of the antenna current to that of the input current, at any given point on the antenna, is affected by these propagation constants. The radiation pattern is derived by integrating the antenna current over the length of the antenna. The antenna current is also affected by the termination impedance at the antenna ends. Two termination impedances which are usually encountered in practice will be considered: (1) the antenna will be terminated in its characteristic impedance to preclude reflected waves; (2) the antenna will be terminated in an open circuit.

Z_o-TERMINATED DIPOLE

When the antenna is terminated in its characteristic impedance, the current in a differential antenna length at a distance, x, from the feed point is (see Figure 1)

$$I\,d\ell = I_{in}\,e^{-\alpha x - j\beta_1 x + j\beta x\cos\phi x}\,dx \qquad (4)$$

and the total summation of the current over the entire antenna is

$$I\ell = \int_0^{\ell_1} I\,d\ell_1 + \int_0^{\ell_2} I\,d\ell_2 = I_{in}\int_0^{\ell_1} e^{-\alpha x - j\left(\beta_1 - \beta\cos\phi x\right)}dx + I_{in}\int_0^{\ell_2} e^{-\alpha x - j\left(\beta_1 + \beta\cos\phi x\right)}dx \qquad (5)$$

performing the integration

$$I\ell = I_{in}\left\{\frac{e^{-\left[a+j\left(\beta_1-\beta\cos\phi\right)\right]\ell_1}-1}{a+j\left(\beta_1-\beta\cos\phi\right)}+\frac{e^{-\left[a+j\left(\beta_1+\beta\cos\phi\right)\right]\ell_2}-1}{a+j\left(\beta_1+\beta\cos\phi\right)}\right\} \tag{6}$$

Equation (6) is applicable to either the θ or ϕ plane and may be substituted into equations (2) and (3) to derive the radiated field strengths in those planes. Substituting equation (6) into equation (2), the radiation field in the elevation plane is

$$E_\phi = \frac{j120\pi I_{in}}{R\lambda}\left(\frac{\sqrt{\frac{\omega\epsilon_o}{j\sigma}}}{\sqrt{\frac{\omega\epsilon_o}{j\sigma}}+\sin\phi}\right)\sin\phi\left\{\frac{1-e^{-\left[a+j\left(\beta_1-\beta\cos\phi\right)\right]\ell_1}}{a+j\left(\beta_1-\beta\cos\phi\right)}+\frac{1-e^{-\left[a+j\left(\beta_1+\beta\cos\phi\right)\right]\ell_2}}{a+j\left(\beta_1+\beta\cos\phi\right)}\right\} \tag{7}$$

and in terms of $a\lambda$ and c/v, which are nearly independent of frequency, the radiation field is

$$E_\phi = j\frac{120\pi I_{in}}{R}\left(\frac{\sqrt{\frac{\omega\epsilon_o}{j\sigma}}\sin\phi}{\sqrt{\frac{\omega\epsilon_o}{j\sigma}}+\sin\phi}\right)\left[\frac{1-e^{-a\lambda\ell_{1\lambda}-j2\pi\ell_{1\lambda}\left(\frac{c}{v}-\cos\phi\right)}}{a\lambda+j2\pi\left(\frac{c}{v}-\cos\phi\right)}+\frac{1-e^{-a\lambda\ell_{2\lambda}-j2\pi\ell_{2\lambda}\left(\frac{c}{v}+\cos\phi\right)}}{a\lambda+j2\pi\left(\frac{c}{v}+\cos\phi\right)}\right] \tag{8}$$

Substituting equation (6) into equation (3), the radiation field in the azimuthal plane is

$$E_\theta = j\frac{120\pi I_{in}\sqrt{\frac{\omega\epsilon_o}{j\sigma}}}{R}\cos\theta\left[\frac{1-e^{-a\lambda\ell_{1\lambda}-j2\pi\ell_{1\lambda}\left(\frac{c}{v}-\cos\theta\right)}}{a\lambda+j2\pi\left(\frac{c}{v}-\cos\theta\right)}+\frac{1-e^{-a\lambda\ell_{2\lambda}-j2\pi\ell_{2\lambda}\left(\frac{c}{v}+\cos\theta\right)}}{a\lambda+j2\pi\left(\frac{c}{v}+\cos\theta\right)}\right] \tag{9}$$

OPEN-TERMINATED DIPOLE

In the case of the open-terminated dipole antenna (see Figure 1), the current along the antenna as derived from transmission line theory is[3]

$$I d\ell_1 = \frac{V_{r_1}}{Z_o}\left\{\sinh\left[\left(\alpha + j\beta_1\right)\left(\ell_1 - x\right) + j\beta x \cos\phi\right]\right\} \tag{10}$$

and the input current is

$$I_{in} = \frac{V_{r_1}}{Z_o}\left[\sinh\left(\alpha + j\beta_1\right)\ell_1\right] \tag{11}$$

Combining equations (10) and (11)

$$I d\ell_1 = I_{in}\left\{\frac{\sinh\left[\left(\alpha + j\beta_1\right)\left(\ell_1 - x\right) + j\beta x \cos\phi\right]}{\sinh\left[\left(\alpha + j\beta_1\right)\ell_1\right]}\right\} \tag{12}$$

The current on the other side of the feed point, $I d\ell_2$, is derived in a similar manner and the summation of the total current over the antenna is

$$\begin{aligned} I\ell &= \int_0^{\ell_1} I d\ell_1 + \int_0^{\ell_2} I d\ell_2 \\ &= I_{in}\left\{\int_0^{\ell_1} \frac{\sinh\left[\left(\alpha + j\beta_1\right)\left(\ell_1 - x\right) + j\beta x \cos\phi\right]}{\sinh\left[\left(\alpha + j\beta_1\right)\ell_1\right]} dx \right. \\ &\quad \left. + \int_0^{\ell_2} \frac{\sinh\left[\left(\alpha + j\beta_1\right)\left(\ell_2 - x\right) - j\beta x \cos\phi\right]}{\sinh\left[\left(\alpha + j\beta_1\right)\ell_2\right]} dx\right\} \end{aligned} \tag{13}$$

If this integral is evaluated and the identity, $\cosh \pm jx = \cos x$, is used,

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[3] Skilling, H. H., "Electric Transmission Lines," McGraw-Hill Book Co., New York, N. Y., 1951, p. 55.

$$\mathcal{U} = -I_{in}\left[\frac{\cos\left(\beta\ell_1\cos\phi\right) - \cosh\left(\alpha + j\beta_1\right)\ell_1}{\left(\alpha + j\beta_1 - j\beta\cos\phi\right)\sinh\left(\alpha + j\beta_1\right)\ell_1} + \frac{\cos\left(\beta\ell_2\cos\phi\right) - \cosh\left(\alpha + j\beta_1\right)\ell_2}{\left(\alpha + j\beta_1 + j\beta\cos\phi\right)\sinh\left(\alpha + j\beta_1\right)\ell_2}\right] \tag{14}$$

It is generally desirable to feed this antenna at the lowest impedance point which is at a current maximum and to operate it at a resonant frequency under those restrictions $\beta_1\ell_1$ and $\beta_1\ell_2 \cong \pi/2,\ 3\pi/2,\ 5\pi/2, \ldots.$ With the use of the identities

$$\cosh\left(x + j\frac{n\pi}{2}\right) = j\left(\sinh x\ \sin\frac{n\pi}{2}\right) \tag{15}$$

and

$$\sinh\left(x + j\frac{n\pi}{2}\right) = j\left(\cosh x\ \sin\frac{n\pi}{2}\right) \tag{16}$$

and

$$\sin\frac{n\pi}{2} = \frac{j}{j^n}$$

where $n = 1, 3, 5, \ldots,$ equation (14) reduces to

$$\mathcal{U} = I_{in}\left\{\frac{\sinh\alpha\ell_1 + j^n\cos\left(\beta\ell_1\cos\phi\right)}{\left[\alpha + j\left(\beta_1 - \beta\cos\phi\right)\right]\cosh\alpha\ell_1} + \frac{\sinh\alpha\ell_2 + j^n\cos\left(\beta\ell_2\cos\phi\right)}{\left[\alpha + j\left(\beta_1 + \beta\cos\phi\right)\right]\cosh\alpha\ell_2}\right\} \tag{17}$$

If the antenna is center-fed, $\ell_1 = \ell_2$ and equation (17) reduces to

$$\mathcal{U} = I_{in}\left[\tanh\alpha\frac{\ell}{2} + j^n\frac{\cos\left(\beta\frac{\ell}{2}\cos\phi\right)}{\cosh\alpha\frac{\ell}{2}}\right]\left[\frac{1}{\alpha + j\left(\beta - \beta_1\cos\phi\right)} + \frac{1}{\alpha + j\left(\beta + \beta_1\cos\phi\right)}\right] \tag{18}$$

IDEALIZED DIPOLE

When the antenna is one-half wavelength long, has no loss, $\alpha = 0$, and the antenna wave velocity is equal to that of free space, equation (18) reduces to

$$\mathcal{U} = I_{in} \frac{\cos\left(\frac{\pi}{2}\cos\phi\right)}{\beta \sin^2\phi} \tag{19}$$

If equation (19) is substituted into equation (3) the radiation field of an idealized antenna is

$$E_\theta = j\frac{120 I_{in}}{R}\sqrt{\frac{\omega\epsilon_o}{j\sigma}}\cos\theta\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right] \tag{20}$$

This agrees with Golden's[4] derivation of the radiation from an idealized dipole.

The radiation fields of the resonant, open-terminated antenna fed at a current maximum can now be written. Substituting $\mathcal{U}$ from equation (17) for $Id\ell$ in equation (2) and rearranging terms, yields the elevation pattern

$$E_\phi = j\frac{120\pi I_{in}}{R}\left(\frac{\sqrt{\frac{\omega\epsilon_o}{j\sigma}}\sin\phi}{\sqrt{\frac{\omega\epsilon_o}{j\sigma}}+\sin\phi}\right)\left\{\frac{\sinh\alpha\lambda\ell_{1\lambda} + j^n\cos\left(2\pi\ell_{1\lambda}\cos\phi\right)}{\left[\alpha\lambda + j2\pi\left(\frac{c}{v} - \cos\phi\right)\right]\cosh\alpha\lambda\ell_{1\lambda}} + \frac{\sinh\alpha\lambda\ell_{2\lambda} + j^n\cos\left(2\pi\ell_{2\lambda}\cos\phi\right)}{\left[\alpha\lambda + j2\pi\left(\frac{c}{v} - \cos\phi\right)\right]\cosh\alpha\lambda\ell_{2\lambda}}\right\} \tag{21}$$

Substituting $\mathcal{U}$ (equation (17)) for $Id\ell$ (equation (3)), yields the azimuthal pattern

[4] Ibid., Golden, R. M. et al, p. 8.

$$E_\theta = j\frac{120\pi I_{in}\sqrt{\frac{\omega\epsilon_o}{j\sigma}}}{R}\cos\theta\left\{\frac{\sinh a\lambda\ell_{1\lambda} + j^n\cos\left(2\pi\ell_{1\lambda}\cos\theta\right)}{\left[a\lambda + j2\pi\left(\frac{c}{v} - \cos\theta\right)\right]\cosh a\lambda\ell_{1\lambda}} + \frac{\sinh a\lambda\ell_{2\lambda} + j^n\cos\left(2\pi\ell_{2\lambda}\cos\theta\right)}{\left[a\lambda + j2\pi\left(\frac{c}{v} + \cos\theta\right)\right]\cosh a\lambda\ell_{2\lambda}}\right\} \tag{22}$$

INPUT IMPEDANCE

The input impedance of the horizontal antenna is determined by the manner in which power is fed to the conductor and by the method in which the ends are terminated. The ends should be terminated with an impedance equal to the conjugate of the characteristic impedance of the conductor for a very large bandwidth. The input impedance is then equal to the characteristic impedance if the antenna is end-fed against a ground plane, or is equal to twice the characteristic impedance if it is center-fed as a dipole. If "n" conductors are used, the input impedance is reduced by a factor of "n."

EXPERIMENTAL INPUT IMPEDANCE

In practice, Z_0 varies from about 250 ohms for a conductor lying on the earth to 600 ohms for a conductor located a few meters above the earth. The measured input impedance of a one-conductor and a five-conductor experimental antenna is shown in Figure 2. The conductors were fed against a ground plane at one end and each was terminated at the other end with an ordinary wirewound resistor. If extremely large bandwidths are not required, the conductors need not be terminated. The input impedance of an antenna with moderate bandwidth, laid on the lava beds of Hawaii, is shown in Figure 3. Five conductors of slightly different lengths were used. A half-power bandwidth of 80 percent was achieved with these unterminated conductors when they were driven with a 50 ohm transmitter. The theoretical input impedance of five equal-length unterminated conductors, elevated 15 ft above the earth, such as those located in the Sierra Nevada Mountains of California[5] is shown in Figure 4. When driven by a 33 ohm transmitter, this antenna has a half-power bandwidth of 20 percent. The radiation efficiency is 12.5 percent at 20 kHz.

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[5] Seeley, E. W., "An Easily Constructed VLF Transmitting Antenna," NOLC Report 669, August 1966.

THEORETICAL INPUT IMPEDANCE

The input impedance of unterminated horizontal antennas can be computed from transmission line equations with a fair degree of accuracy. If the antenna is not end-fed, the two portions on either side of the feed point can be treated as two transmission lines and their impedances can be added to obtain the total input impedance. If a conductor is end-fed against a ground plane of negligible resistance, the input impedance is

$$Z_{in} = Z_o \coth(\alpha \ell + j\beta \ell) \tag{23}$$

It can easily be shown from transmission line theory that the characteristic impedance is

$$Z_o = \frac{\beta_1 - j\alpha}{\omega C_p} = \frac{2\pi \frac{c}{v} - j\alpha\lambda}{2\pi C_p c} \tag{24}$$

The input impedance can now be expressed in terms which are almost independent of frequency over the VLF band (with the exception of the length term), and is

$$Z_{in} C_p = \frac{2\pi \frac{c}{v} - j\alpha\lambda}{2\pi c} \coth\left(\alpha\lambda + j2\pi\frac{c}{v}\right)\ell_\lambda \tag{25}$$

This equation is applicable to an antenna with any number of parallel conductors provided their separation is at least 3.5 skin depths. The capacitance to earth, C_p, is "n" times greater for "n" conductors, which reduces Z_{in} by a factor of "n" as compared to a single conductor.

The normalized input impedance (equation (25)), has been plotted as a function of antenna length for typical values of antenna wave velocity and attenuation factors normally found in practical antenna design. Figures 5 through 8 are plots of input impedance magnitude and phase angle.

ATTENUATION AND WAVE VELOCITY

The wave-velocity ratio and attenuation-wavelength product can be derived from the transmission line equations for the propagation constants

$$\alpha + j\beta_1 = \sqrt{(r + j\omega L)j\omega C_p} \tag{26}$$

where the conductance to ground is negligible, as is usually the case at VLF. Expressions for $\alpha\lambda$ and c/v are easily derived from equation (26) and are

$$\alpha\lambda = \frac{c}{f}\sqrt{\omega r C_p}\,\sqrt[4]{1 + Q^2}\,\cos\left(\frac{1}{2}\,\text{arc tan} - \frac{1}{Q}\right) \tag{27}$$

$$\frac{c}{v} = \frac{c}{\omega}\sqrt{\omega r C_p}\,\sqrt[4]{1 + Q^2}\,\sin\left(\frac{1}{2}\,\text{arc tan} - \frac{1}{Q}\right) \tag{28}$$

where the angle (arc tan - 1/Q) is in the second quadrant and $Q = \omega L/r$.

The antenna series inductance, series resistance, and shunt capacitance can be computed from the physical dimensions of the antenna, provided the earth conductivity is known. Carson[6] has derived equations for the series inductance and resistance of a conductor over the earth. In the case where the antenna height is very much less than the skin depth the inductance/unit length is

$$L = \frac{\mu}{2\pi}\,\ln\frac{\delta\sqrt{2}}{a} \tag{29}$$

and the resistance/unit length is

$$r = \pi^2 f \times 10^{-7} + R_{ac} + R_m \tag{30}$$

where the skin depth is

$$\delta = \frac{503}{\sqrt{f\sigma}}\ \text{meters} \tag{31}$$

and

$$R_{ac} = \frac{1.72 \times 10^{-8} f}{a\sqrt{0.172f} - 0.0137} \tag{32}$$

At VLF the capacitance to earth is obtained from the solution of Laplace's equation and is

$$C_p = \frac{2\pi\epsilon_o}{\ln\dfrac{h + \sqrt{h^2 - a^2}}{a}} \tag{33}$$

[6] Carson, J. R., "Wave Propagation in Overhead Wires With Ground Return," Bell System Tech. Jour., Vol. 5, 1926, pp. 539-554.

$$C_p = \frac{2\pi\epsilon_o}{\ell n \frac{2h}{a}} \qquad \text{when } h \gg a \tag{34}$$

The wave-velocity ratio and attenuation-wavelength product has been computed and plotted as a function of frequency in Figures 9 through 14 for a typical-sized conductor located at various heights above the earth and having two widely different conductivities. As shown in these figures, there is very little variation in the wave velocity over a wide frequency range. The attenuation-wavelength product is also quite insensitive to frequency, except in the case of small diameter conductors at low frequencies, where the ac resistance increases rapidly. The earth's conductivity does not have a significant influence on $\alpha\lambda$ and c/v, as indicated in Figure 15. The conductor size is not very important unless the conductor is very near the ground. The curves shown are for 20 kHz bandwidth but are a fair approximation for the entire VLF band.

RESONANT RESISTANCE OF LOW LOSS ANTENNAS

It is essential in computing efficiency to determine the lowest resonant input resistance of the antenna. This low occurs at positions of current maximum along the antenna length. At the current maximum $\beta_1 \ell_n = \pi/2, 3\pi/2, 5\pi/2, \ldots$, where ℓ_n is the distance from current maximum to the end of the conductor. Then equation (23) reduces to

$$Z_{in} = Z_o \tanh \alpha \ell \tag{35}$$

For a properly designed horizontal antenna the value of Q is in the order of 10. When $Q \gg 1.0$, equation (27) reduces to

$$\alpha = \frac{r}{2Z_o} \tag{36}$$

Combining equations (35) and (36)

$$Z_{in} = Z_o \tanh \frac{r\ell}{2Z_o} \tag{37}$$

Antennas with Q values in the order of 10 closely approximate resistive characteristic impedance. If the antenna is short enough so that $\alpha\ell \ll 1$, a very close approximation to the input impedance at a current maximum is

$$R_{oin} = \frac{r\ell}{2} \tag{38}$$

This very simple expression for the input impedance at a current maximum holds true for any resonant length within the restricted values of Q and attenuation cited above.

THE LOSSY LENGTHENING FACTOR

Antennas in free space resonate at lengths which are near multiples of one-half wavelength. However, a rigorous analysis of the input impedance of a dipole near the earth shows that resonance (resistive input impedance) occurs when the antenna wavelength is something other than a multiple of one-half wavelength. This phenomenon is due to antenna losses which cause the characteristic impedance to become complex. The antenna losses will cause the resonant lengths to be slightly greater than odd multiples and slightly less than even multiples of one-half wavelength on the antenna. In fact, if the antenna losses become too great the antenna will not resonate at any length.

The normalized input impedance of a center-fed dipole near the earth as derived from equations (23) and (24) is

$$Z_{in}\pi f C_p = \left(\beta_1 - j\alpha\right)\coth\left(\frac{\alpha\ell}{2} + j\frac{\beta_1\ell}{2}\right) \tag{39}$$

Let

$$\coth\left(\frac{\alpha\ell}{2} + j\frac{\beta_1\ell}{2}\right) = R_e + jI_m \tag{40}$$

If the left side of equation (40) is expanded into a real and imaginary term, and the real and imaginary terms from each side of equation (40) are equated respectively, the ratio of the real terms to the imaginary terms is

$$\frac{R_e}{I_m} = \frac{\sinh\alpha\ell}{-\sin\beta_1\ell} \tag{41}$$

Substituting equation (40) into (39) yields the normalized input impedance

$$\begin{aligned} Z_{in}\pi f C_p &= \left(\beta_1 - j\alpha\right)\left(R_e + jI_m\right) \\ &= \left(\beta_1 R_e + \alpha I_m\right) + j\left(\beta_1 I_m - \alpha R_e\right) \end{aligned} \tag{42}$$

The input impedance is resistive, or resonance occurs, when

$$\beta_1 I_m = \alpha R_e$$

or (43)

$$\frac{R_e}{I_m} = \frac{\beta_1}{\alpha}$$

Combining equations (41) and (43), the relationship between α, β_1, and ℓ at resonance is

$$\frac{\beta_1}{\alpha} = \frac{\sinh \alpha \ell}{-\sin \beta_1 \ell} \tag{44}$$

Equation (44) is more useful if it is written in terms of the Q of the conductor. It can easily be shown from equations (27) and (28) that

$$\frac{\beta_1}{\alpha} = Q + \sqrt{1 + Q^2} \tag{45}$$

If equation (45) is substituted into equation (44) the condition for resonance is

$$\left(Q + \sqrt{1 + Q^2}\right) \sin \beta_1 \ell = \sinh\left(\frac{\beta_1 \ell}{Q + \sqrt{1 + Q^2}}\right) \tag{46}$$

This equation is solved and the dipole electrical length for resonance is plotted as a function of Q in Figure 16. The results shown apply generally to all lossy dipoles.

HORIZONTAL ANTENNA EFFICIENCY

The efficiency of the horizontal antenna can best be derived by comparing its radiated field strength with that of a perfect vertical monopole located over a perfectly conductive earth. The radiated field strength of a top-loaded monopole in the horizontal plane is

$$E_f = \frac{120\pi I h}{R\lambda} \tag{47}$$

and the radiation resistance for such an antenna is[7]

$$R_r = 160\pi^2\left(\frac{h}{\lambda}\right)^2 \tag{48}$$

The radiated power is

$$P_r = I^2R_r \tag{49}$$

Substituting equation (48) into (49), solving for I, and substituting the results into equation (47) gives an expression for the radiated field strength in terms of radiated power

$$E_f = \frac{\sqrt{90P_r}}{R} \tag{50}$$

The radiated field of a horizontal conductor near the earth, in the azimuthal plane is

$$d\,E_\theta = j\frac{120\pi}{R\lambda}(I\,d\ell)\sqrt{\frac{\omega\epsilon_o}{j\sigma}}\cos\theta \tag{3}$$

If equation (50) is now equated to equation (3), the power radiated by a horizontal conductor in comparison to that radiated by a perfect monopole is

$$P_r = \frac{160\pi^2\omega\epsilon_o}{\sigma}\cos^2\theta\left(\frac{I\,d\ell}{\lambda}\right)^2 \tag{51}$$

The efficiency, defined as the ratio of radiated power to input power, is

$$\frac{P_r}{P_{in}} = \eta = \frac{160\pi^2\omega\epsilon_o}{\sigma Z_{in}}\cos^2\theta\left(\frac{I\,d\ell}{\lambda I_{in}}\right)^2 \tag{52}$$

Equations for the efficiency of horizontal antennas near the earth with various termination and feed point positions can now be derived.

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[7] Jasik, H., "Antenna Engineering Handbook," First Edition, McGraw-Hill Book Co., Inc., New York, N. Y., 1961, p. 19-6.

Z_o-TERMINATED DIPOLE

The efficiency of a dipole near the earth, terminated in its characteristic impedance and fed at any position along its length, is derived by combining equations (6) and (52), and is

$$\eta = \frac{80\pi^2\omega\epsilon_o}{\sigma Z_o \lambda^2}\cos^2\theta\left[\frac{1 - e^{-\alpha\ell_1 - j\left(\beta_1 - \beta\cos\theta\right)\ell_1}}{\alpha + j\left(\beta_1 - \beta\cos\theta\right)} + \frac{1 - e^{-\alpha\ell_2 - j\left(\beta_1 + \beta\cos\theta\right)\ell_2}}{\alpha + j\left(\beta_1 + \beta\cos\theta\right)}\right]^2 \tag{53}$$

where $Z_{in} = 2Z_o$.

The efficiency may be expressed in terms of antenna length in wavelengths, frequency, and certain parameters that are nearly independent of frequency over the VLF band. If equation (24) is substituted into equation (53) with the terms rearranged and the propagation constants restated in terms of the attenuation-wavelength product, $\alpha\lambda$, and the wave-velocity ratio, c/v, the efficiency is

$$\eta = \frac{\frac{8}{3}\pi^3 f C_p \cos^2\theta}{\sigma\left(2\pi\frac{c}{v} - j\alpha\lambda\right)}\left[\frac{1 - e^{-\alpha\lambda\ell_{1\lambda} - j2\pi\ell_{1\lambda}\left(\frac{c}{v} - \cos\theta\right)}}{\alpha\lambda + j2\pi\left(\frac{c}{v} - \cos\theta\right)} + \frac{1 - e^{-\alpha\lambda\ell_{2\lambda} - j2\pi\ell_{2\lambda}\left(\frac{c}{v} + \cos\theta\right)}}{\alpha\lambda + j2\pi\left(\frac{c}{v} + \cos\theta\right)}\right] \tag{54}$$

It may be desirable to radiate most of the power in one general direction. In this case the antenna should be end-fed against a ground plane, as is the Beverage wave antenna. The input impedance is then equal to the characteristic impedance if the ground plane resistance is negligible. Only the first term within the brackets in equation (54) remains since $\ell_2 = 0$. The efficiency is

$$\eta = \frac{\frac{16}{3}\pi^3 f C_p \cos^2\theta}{\sigma\left(2\pi\frac{c}{v} - j\alpha\lambda\right)}\left[\frac{1 - e^{-\alpha\lambda\ell_{\lambda} - j2\pi\ell_{\lambda}\left(\frac{c}{v} - \cos\theta\right)}}{\alpha\lambda + j2\pi\left(\frac{c}{v} - \cos\theta\right)}\right]^2 \tag{55}$$

If the efficiency is normalized to frequency, capacitance/unit length (C_p), and conductivity of the earth (σ), a design equation that is very useful results

$$\frac{\eta\sigma}{fC_p} = \frac{165.4}{2\pi\frac{c}{v} - ja\lambda}\left[\frac{1 - e^{-a\lambda\ell_\lambda - j2\pi\ell_\lambda\left(\frac{c}{v}-1\right)}}{a\lambda + j2\pi\left(\frac{c}{v} - 1\right)}\right] \qquad (56)$$

This is the efficiency in a direction off the antenna end, where $\theta = 0°$. By using equation (56), design curves have been prepared for the ranges of parameter values found in practice. The normalized efficiency has been plotted as a function of antenna length for a wide range of wave-velocity ratios (c/v) and attenuation-wavelength products ($a\lambda$), in Figures 17 through 26. In designing the horizontal antenna for a desired efficiency at a certain frequency, the conductivity of the earth under the antenna must be measured. All the other variables in equation (56) can be measured or computed fairly accurately from the physical dimensions of the antenna (see Figures 9 through 13).

OPEN-TERMINATED DIPOLE

The horizontal dipole near the earth is most efficient when placed over low conductivity earth. However, it is quite difficult to construct a low-impedance termination ground plane antenna in low conductivity soil. In many instances it is desirable, if not imperative, to design an open-terminated antenna. The efficiency of this type of antenna is derived by combining equations (14), (23), (24), and (52), and then reducing and rearranging parameters in terms of $a\lambda$ and c/v. The efficiency for any length or feed point position is

$$\eta = \frac{16\pi^3 fC_p \cos^2\theta}{3\sigma\left(2\pi\frac{c}{v} - ja\lambda\right)\left[\coth\left(a\lambda\ell_{1\lambda} + j2\pi\frac{c}{v}\ell_{1\lambda}\right) + \coth\left(a\lambda\ell_{2\lambda} + j2\pi\frac{c}{v}\ell_{2\lambda}\right)\right]}$$

$$\left\{\frac{\cosh\left(a\lambda\ell_{1\lambda} + j2\pi\frac{c}{v}\ell_{1\lambda}\right) - \cos\left(2\pi\ell_{1\lambda}\cos\theta\right)}{\left[a\lambda + j2\pi\left(\frac{c}{v} - \cos\theta\right)\right]\sinh\left(a\lambda\ell_{1\lambda} + j2\pi\frac{c}{v}\ell_{1\lambda}\right)} \qquad (57)$$

$$+ \frac{\cosh\left(a\lambda\ell_{2\lambda} + j2\pi\frac{c}{v}\ell_{2\lambda}\right) - \cos\left(2\pi\ell_{2\lambda}\cos\theta\right)}{\left[a\lambda + j2\pi\left(\frac{c}{v} + \cos\theta\right)\right]\sinh\left(a\lambda\ell_{2\lambda} + j2\pi\frac{c}{v}\ell_{2\lambda}\right)}\right\}^2$$

This equation is simplified somewhat when the antenna is operated at resonance, a condition which facilitates impedance matching and increases bandwidth. When $(c/v)\ell_{1\lambda}$ and $(c/v)\ell_{2\lambda}$ are both equal to either like or unlike odd multiples of $\frac{1}{4}$, the antenna is resonant and is fed at a current maximum. This is generally true only for a lossless antenna but as shown in Figure 16, it is a good approximation for antennas with $Q > 2$. The antenna efficiency is then

$$\eta = \frac{16\pi^3 f C_p \cos^2\theta}{3\sigma\left(2\pi\frac{c}{v} - ja\lambda\right)\left(\tanh a\lambda\ell_{1\lambda} + \tanh a\lambda\ell_{2\lambda}\right)} \left\{\frac{\sinh a\lambda\ell_{1\lambda} + j^n \cos\left(2\pi\ell_{1\lambda}\cos\theta\right)}{\left[a\lambda + j2\pi\left(\frac{c}{v} - \cos\theta\right)\right]\cosh a\lambda\ell_{1\lambda}} + \frac{\sinh a\lambda\ell_{2\lambda} + j^n \cos\left(2\pi\ell_{2\lambda}\cos\theta\right)}{\left[a\lambda + j2\pi\left(\frac{c}{v} + \cos\theta\right)\right]\cosh a\lambda\ell_{2\lambda}}\right\}^2 \qquad (58)$$

where $n = 1, 3, 5, 7, \ldots$.

In some cases it may be advantageous to feed the antenna at one end against a very low impedance ground plane. The second term containing $\ell_{2\lambda}$ (inside the braces) in equation (57) vanishes; in addition, that portion of the input impedance caused by ℓ_2 is equal to zero. The efficiency is then

$$\eta = \frac{16\pi^3 f C_p \cos^2\theta}{3\sigma\left(2\pi\frac{c}{v} - ja\lambda\right)\coth\left(a\lambda\ell_\lambda + j2\pi\frac{c}{v}\ell_\lambda\right)} \left\{\frac{\cosh\left(a\lambda\ell_\lambda + j2\pi\frac{c}{v}\ell_\lambda\right) - \cos\left(2\pi\ell_\lambda\cos\theta\right)}{\left[a\lambda + j2\pi\left(\frac{c}{v} - \cos\theta\right)\right]\sinh\left(a\lambda\ell_\lambda + j2\pi\frac{c}{v}\ell_\lambda\right)}\right\}^2 \qquad (59)$$

The efficiency of the end-fed antenna near resonance, where $(c/v)\ell_\lambda = n/4$, and n is odd integers, can be derived from equation (59) with the aid of the identities in equations (15) and (16) and

$$2\sinh x\cosh x = \sinh 2x \qquad (60)$$

The efficiency is

$$\eta = \frac{32\pi^3 f C_p \cos^2\theta}{3\sigma\left(2\pi\frac{c}{v} - j\alpha\lambda\right)\sinh\left(\frac{n\alpha\lambda}{2\frac{c}{v}}\right)}\left[\frac{\sinh\left(\frac{n\alpha\lambda}{4\frac{c}{v}}\right) + j^n\cos\left(\frac{n\pi}{2\frac{c}{v}}\cos\theta\right)}{\alpha\lambda + j2\pi\left(\frac{c}{v} - \cos\theta\right)}\right]^2 \tag{61}$$

MULTIPLE PARALLEL CONDUCTORS

The efficiency of a single conductor near the earth is very low unless it is several wavelengths long, the attenuation is also very low, and the wave velocity is near that of free space wave velocity. However, the efficiency can be increased "n" folds by using "n" parallel conductors, provided the spacing between conductors is great enough to minimize mutual impedance.

The ratio of mutual impedance to self-impedance, taken from Carson,[6] is shown in Figure 27. The mutual impedance reduces the antenna efficiency in four ways: (1) The mutual resistance raises the input resistance; (2) the mutual resistance increases the current attenuation along the antenna; (3) the mutual reactance slows down the wave velocity; and (4) the mutual reactance raises the characteristic impedance. These all have a deteriorating effect on efficiency. The mutual impedance has a much greater effect on efficiency if the antenna is open-terminated rather than being terminated in its characteristic impedance. The mutual resistance has a greater effect on efficiency than mutual reactance (see Figure 27). The resonant input resistance of an open-terminated antenna is nearly one-half the sum of the total self- and mutual-resistance along the conductor. If the conductor spacing is small compared to the skin depth, the self-resistance and mutual resistance are approximately equal (see Figure 27) and very little increase in efficiency results in using two conductors instead of one. A spacing of 3.5 skin depths reduces the mutual resistance to 10 percent of the self-resistance. This spacing represents 0.03 λ in low conductivity areas such as the Hawaiian lava bed. The input impedance of a Z_0-terminated antenna is equal to Z_0. The characteristic impedance is much larger than the self-resistance, and unless the antenna is very long the mutual resistance has very little effect on the input impedance of the Z_0-terminated antenna. The characteristic impedance is proportional to the square root of the series reactance; therefore, the mutual reactance will affect the input impedance of the Z_0-terminated antenna. As is shown in Figure 27, the mutual reactance is very small when compared with the self-reactance except when the conductors are very closely spaced. The mutual reactance is nearly negligible when conductor spacing is equal to one-half the skin depth. Apparently, the input impedance will be halved and the efficiency doubled

for two conductors quite close together (in comparison with that of one conductor) if they are terminated in their characteristic impedance.

The current attenuation along the antenna resulting from mutual resistance has about the same deteriorating effect on efficiency regardless of the type of antenna termination. This effect is quite small, except in the case of very long antennas.

Mutual reactance reduces wave velocity along the antenna, which results in reduced efficiency for long antennas. This is not a serious problem since series capacitors can be inserted in the antenna to increase the wave velocity.

Because the induction field coupling between conductors is weak, but the radiation field coupling is strong, "n" parallel conductors will radiate "n" times as much power as a single conductor. The conductors appear as "n" separate antennas to the induction field and the input resistance is reduced by a factor of "n"; however, to the radiation field the "n" conductors appear as a single conductor and the radiation resistance is not changed by the addition of parallel conductors. Since the same current flows through the input resistance and the radiation resistance, the radiation efficiency is therefore the ratio of the radiation resistance to the input resistance. Since the input resistance is reduced by a factor of "n" by "n" conductors, it is readily apparent that the efficiency must be increased by a factor of "n."

CLOSELY SPACED PARALLEL CONDUCTORS

There is another characteristic of the horizontal-conductor antenna near the earth that is not readily apparent: with the same total input power, "n" antennas closely spaced (0.03λ) will radiate "n" times as much power as "n" antennas spaced one-half wavelength apart. It is apparent that the closely spaced conductors must radiate more power that the $\lambda/2$ spaced conductors if the total antenna current is identical in both cases. The resulting maximum radiated field strengths will be identical in both cases; however, the $\lambda/2$ spaced conductors will have a much narrower radiation pattern than the closely spaced conductors. The reason for this phenomenon is shown in Figure 28. If "n" identical short antennas (spaced $\lambda/2$ apart) are excited by identical power sources the current in each antenna is

$$I = \sqrt{\frac{P_{in}}{R_{in}}} \qquad \text{where } R_{in} \gg R_r \qquad (62)$$

The radiated power from "n" antennas is then

$$P_r = nI^2 R_r = \frac{nP_{in} R_r}{R_{in}} \tag{63}$$

The radiated field strength of the antenna array shown in Figure 28 is

$$E_f = KI \cos\theta \frac{\sin\left(n\frac{\pi}{2}\sin\theta\right)}{\sin\left(\frac{\pi}{2}\sin\theta\right)} \tag{64}$$

and has a maximum value of

$$E_{f_{max}} = Kn\sqrt{\frac{P_{in}}{R_{in}}} \tag{65}$$

when $\theta = 0°$.

If these same antennas are brought close together as in Figure 28(b), are connected in parallel, and excited by the same total power as the array in equation (65), the antenna current is

$$I = n\sqrt{\frac{P_{in}}{R_{in}}} \tag{66}$$

and the radiated power of the closely spaced conductors is

$$P_r = \left(n\sqrt{\frac{P_{in}}{R_{in}}}\right)^2 R_r = n^2 \frac{P_{in}}{R_{in}} R_r \tag{67}$$

The radiated field strength shown in Figure 28(b) is

$$E_f = KI \cos\theta \tag{68}$$

and has a maximum value at $\theta = 0°$ of

$$E_{f_{max}} = Kn\sqrt{\frac{P_{in}}{R_{in}}} \tag{69}$$

The maximum field strengths of the $\lambda/2$ spaced antennas and the closely spaced antennas are identical (see equations (65) and (69)); however, the closely spaced antennas radiate "n" times more power than the $\lambda/2$ spaced

antennas (see equations (63) and (67)). This is a characteristic of lossy antennas whose loss resistance is much greater than their radiation resistance, and whose conductors are coupled by the radiation field but isolated from their individual induction fields.

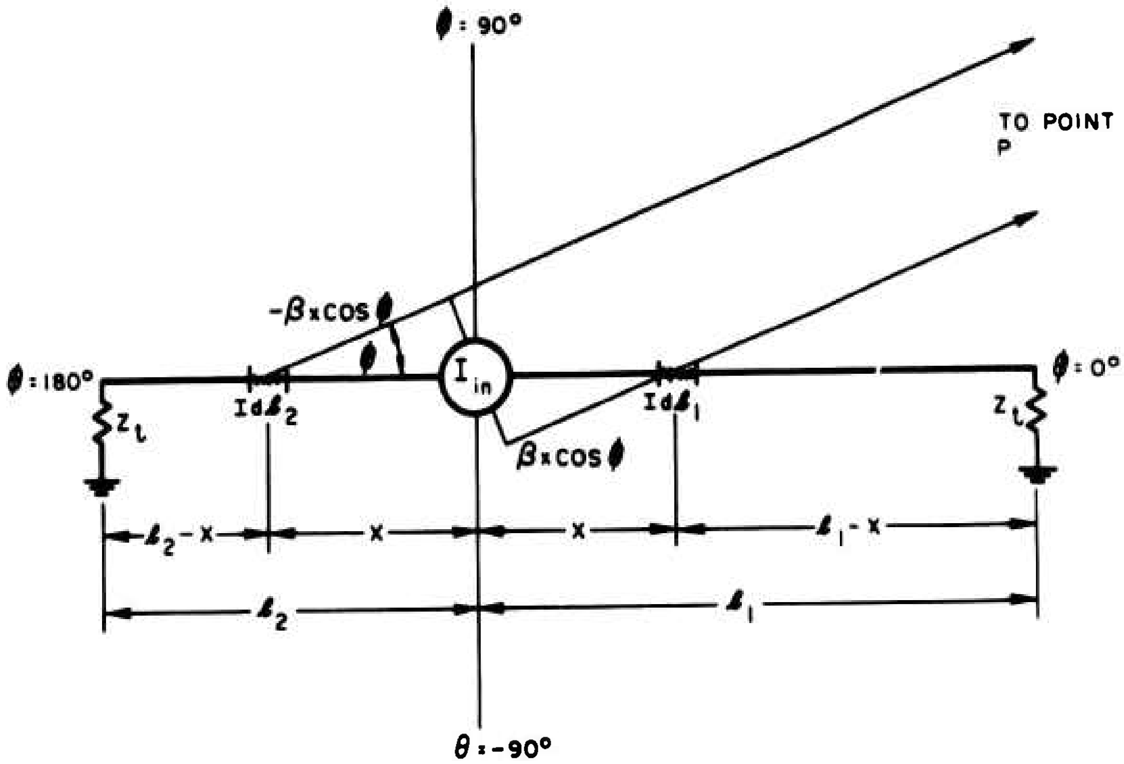


FIGURE 1. A Radiating VLF Conductor Near Earth

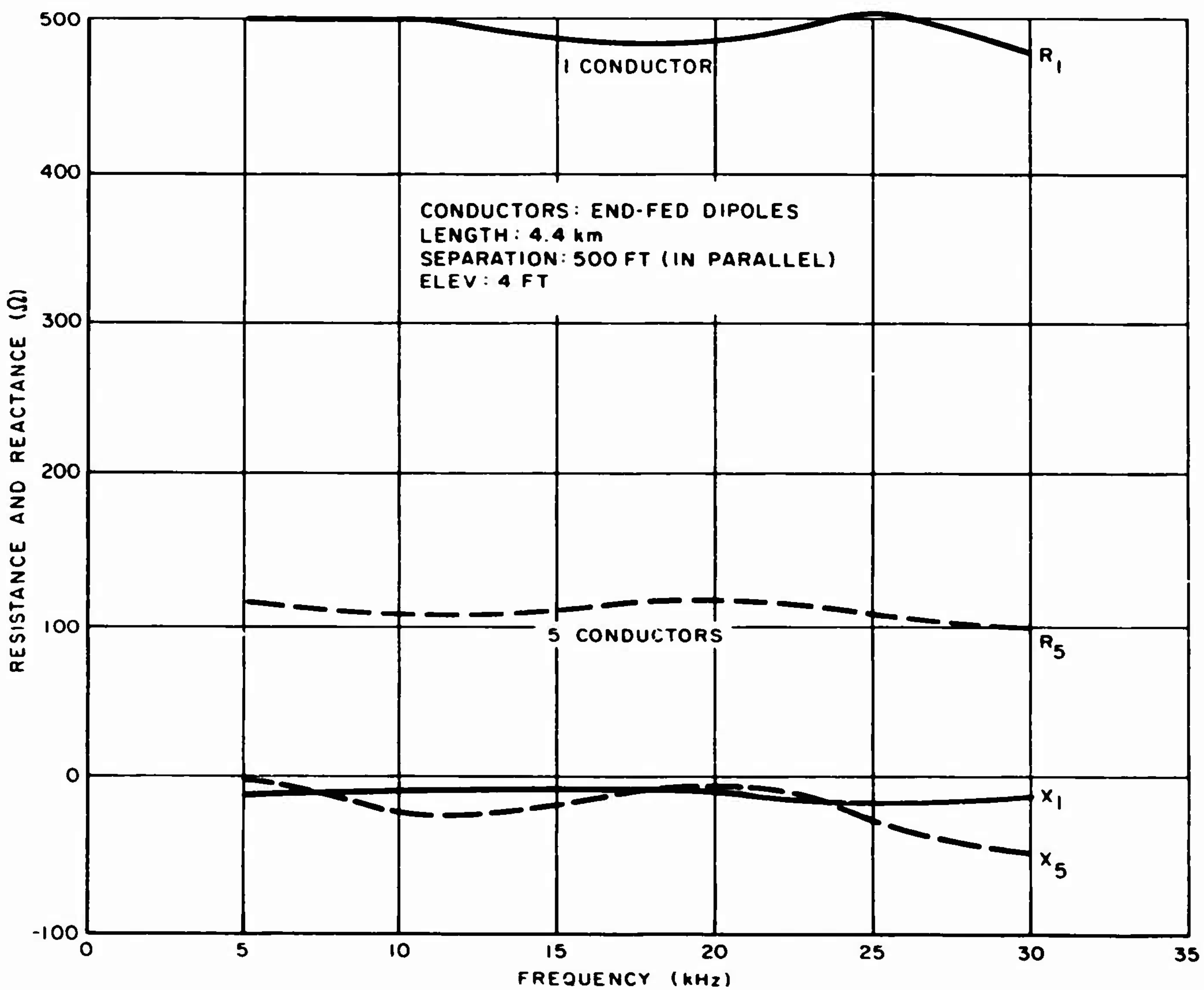


FIGURE 2. Input Impedance of California Lava Bed Antenna

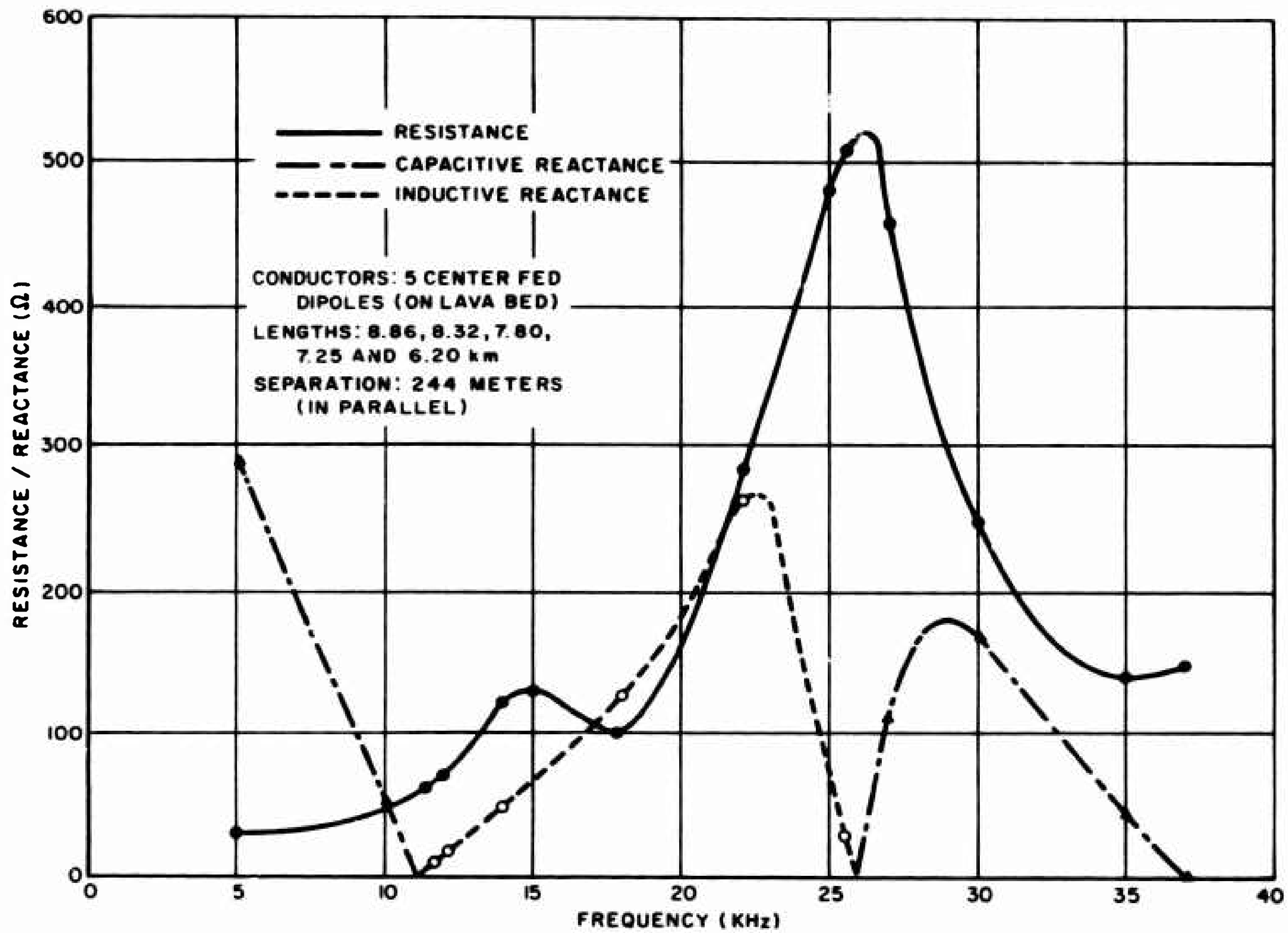


FIGURE 3. Input Impedance of Hawaiian Island Lava Bed Antenna

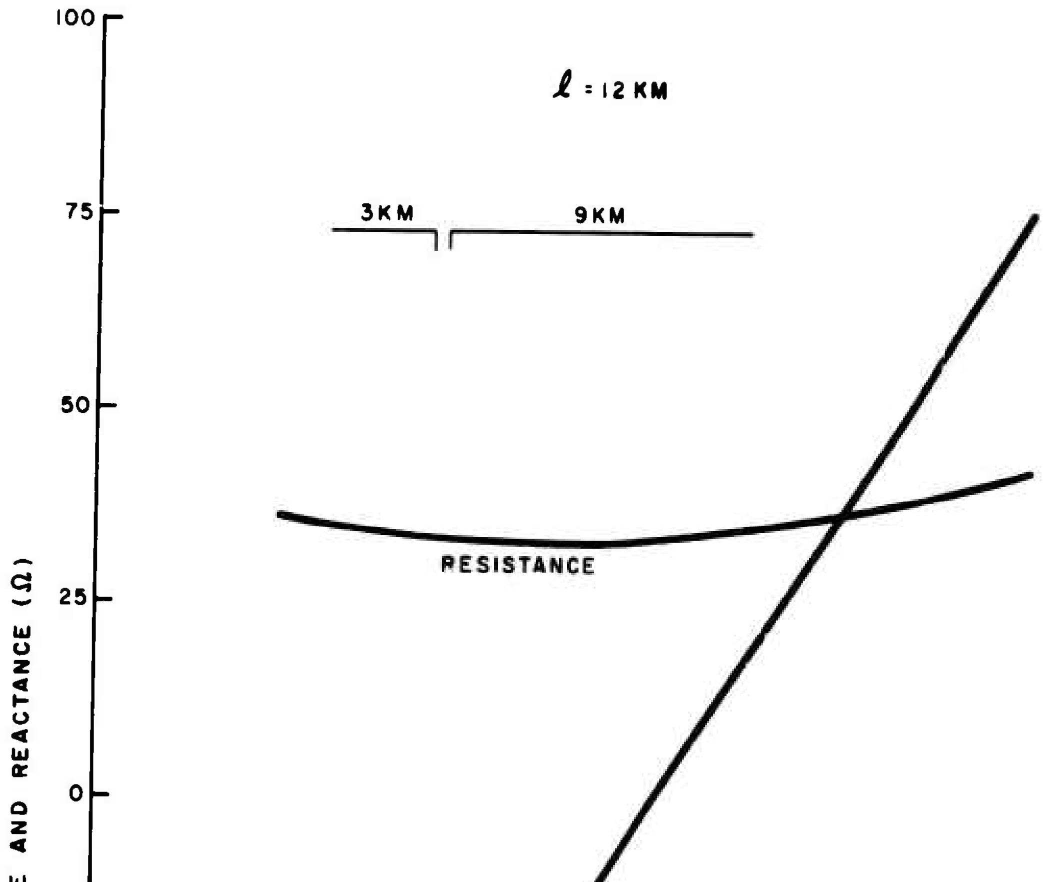


FIGURE 4. Theoretical Input Impedance of Sierra Nevada Antenna

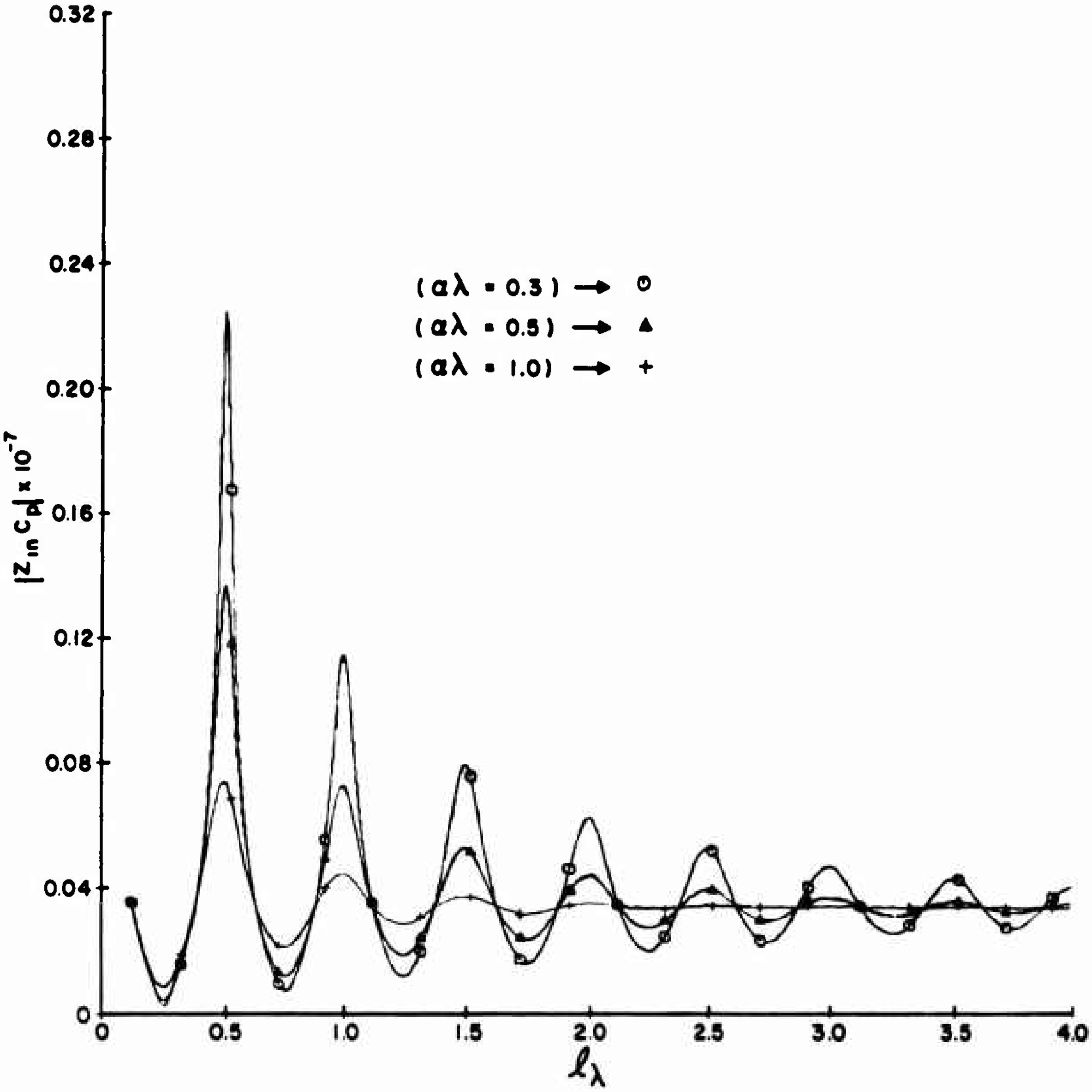


FIGURE 5. Normalized Input Impedance of End-Fed Conductor Near Earth (c/v = 1.0)

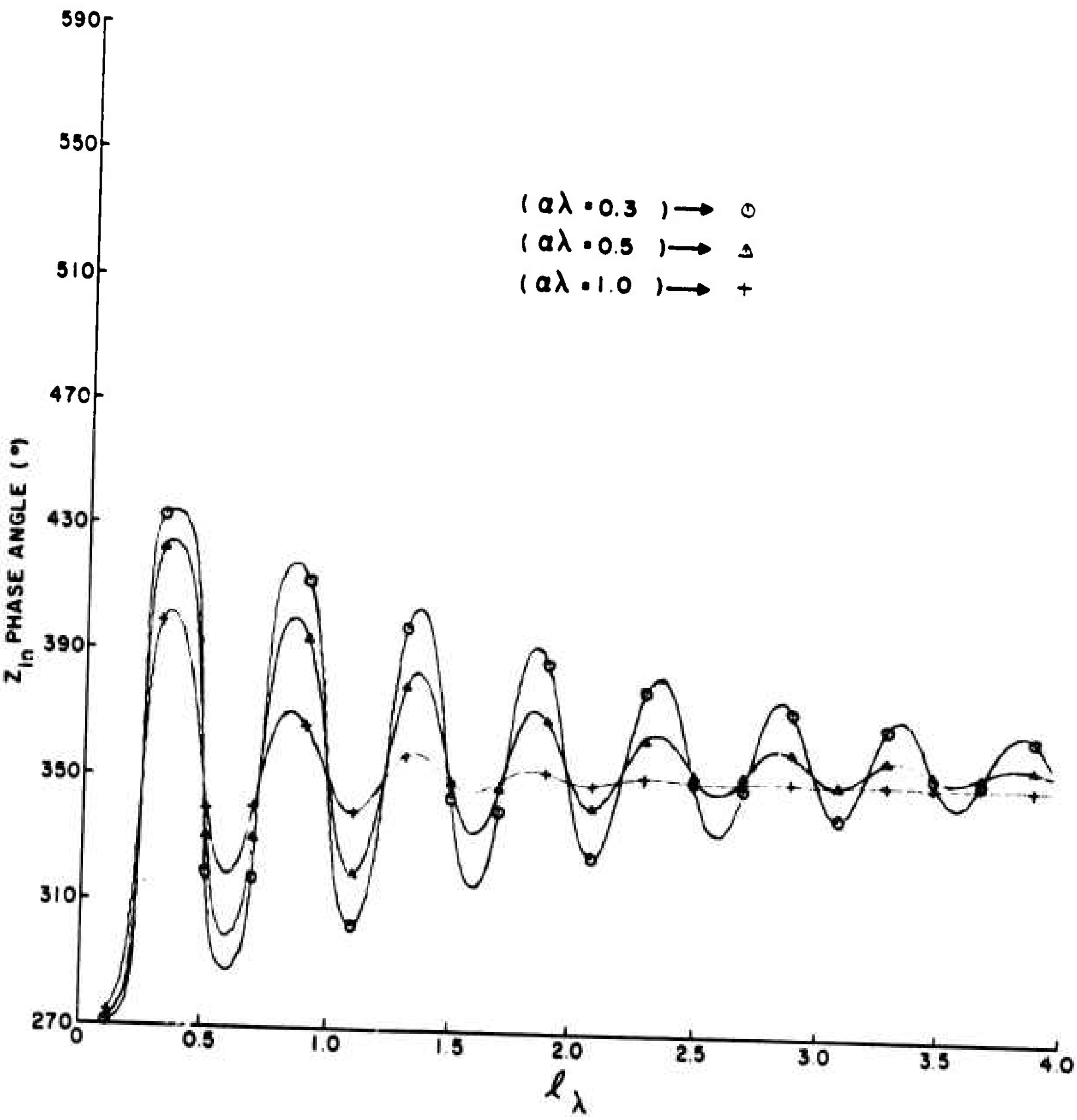


FIGURE 6. Input Impedance Phase Angle of End-Fed Conductor Near Earth (c/v = 1.0)

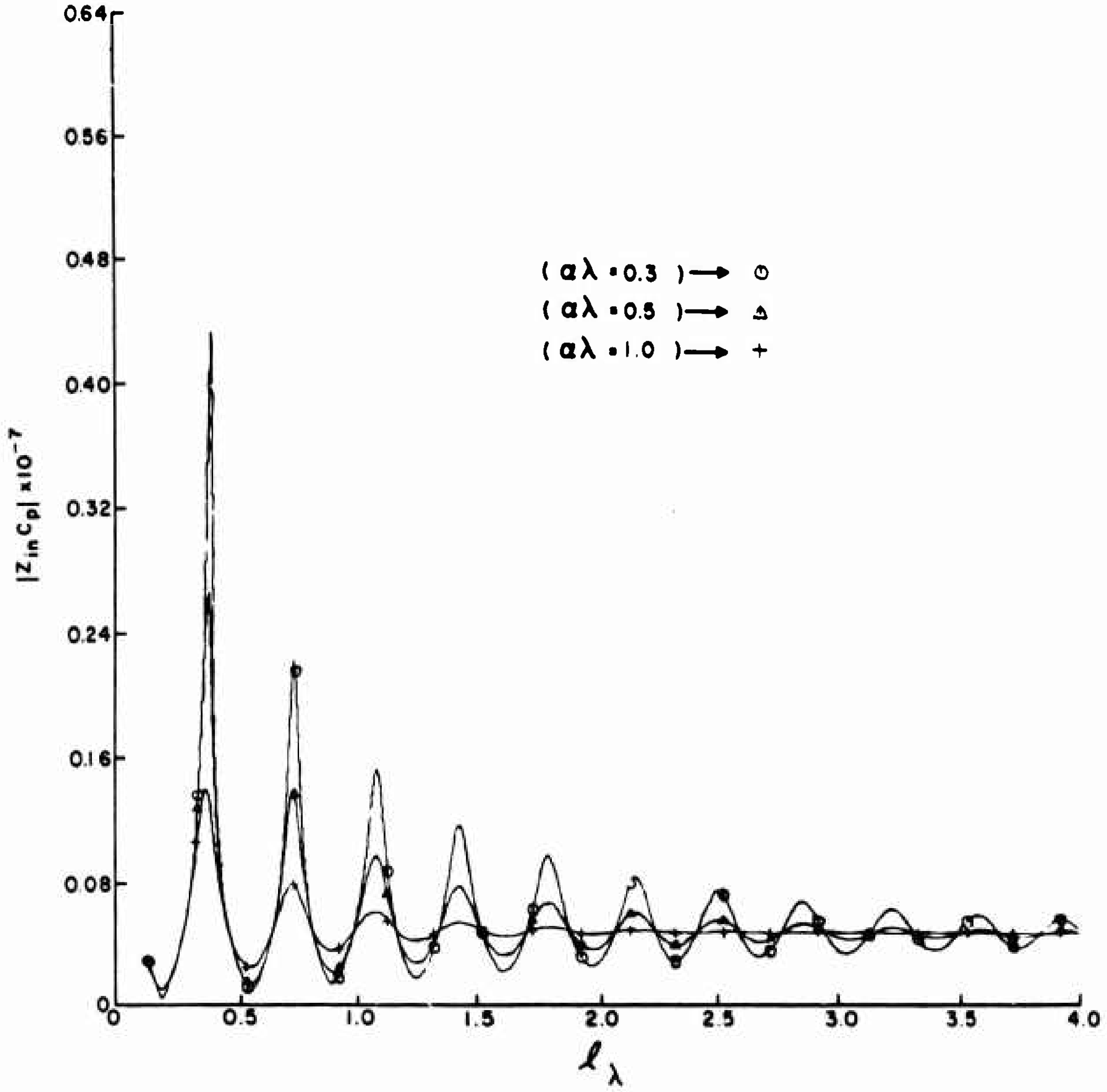


FIGURE 7. Normalized Input Impedance of End-Fed Conductor Near Earth (c/v = 1.4)

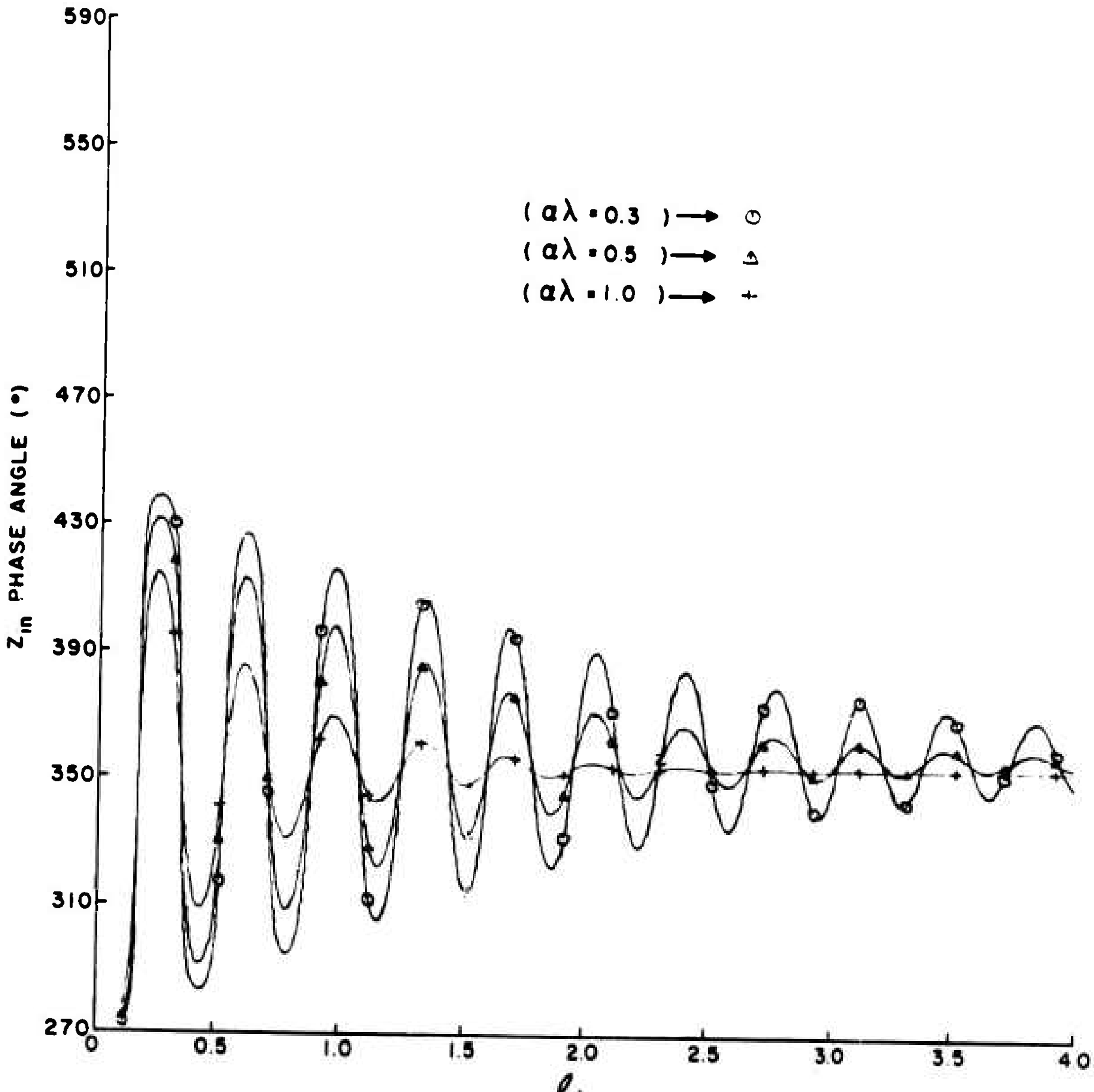


FIGURE 8. Input Impedance Phase Angle of End-Fed Connector Near Earth (c/v = 1.4)

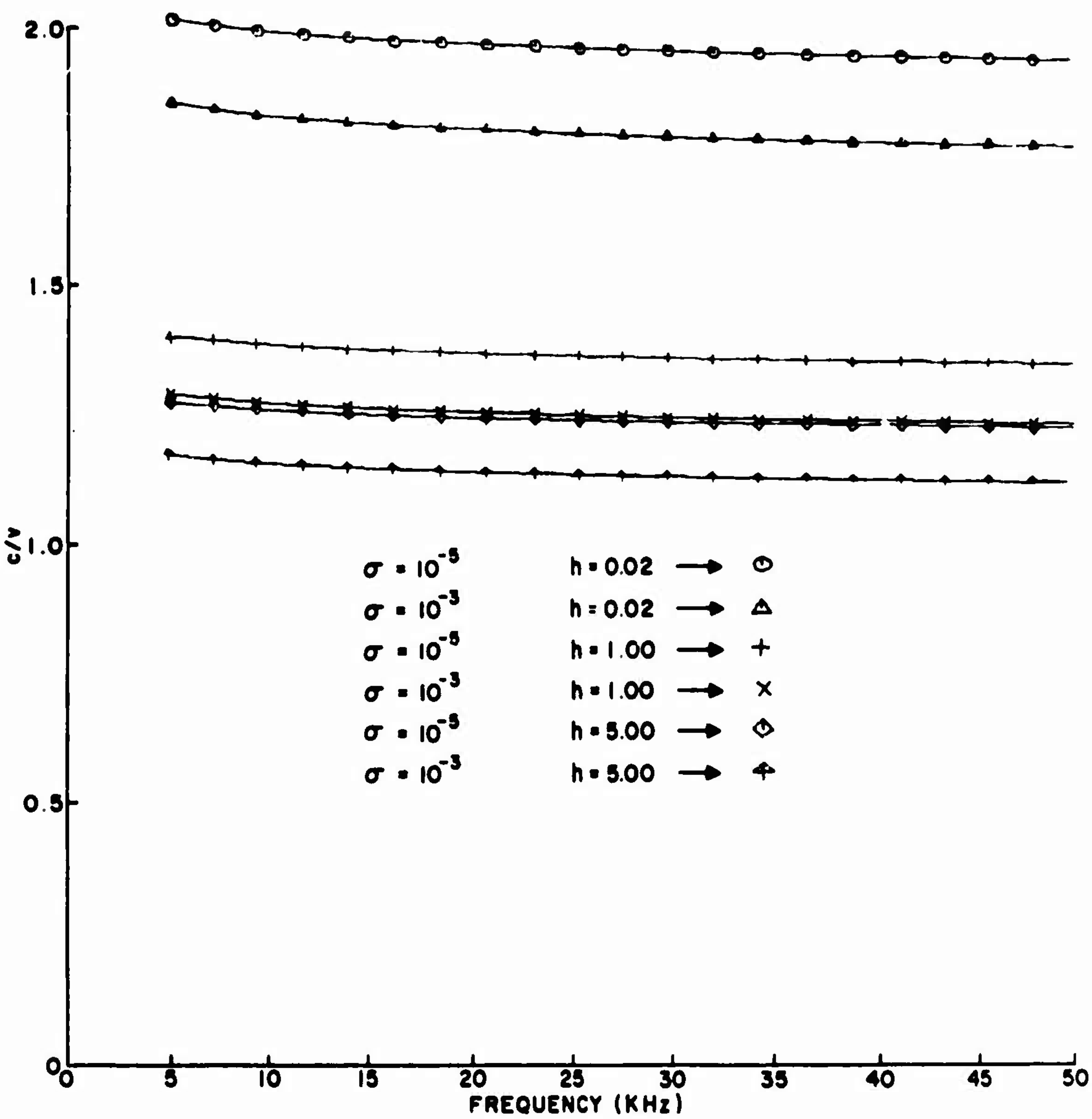


FIGURE 9. Wave Velocity Along Conductor Near Earth
(No. 12 wire radius = 0.1 cm)

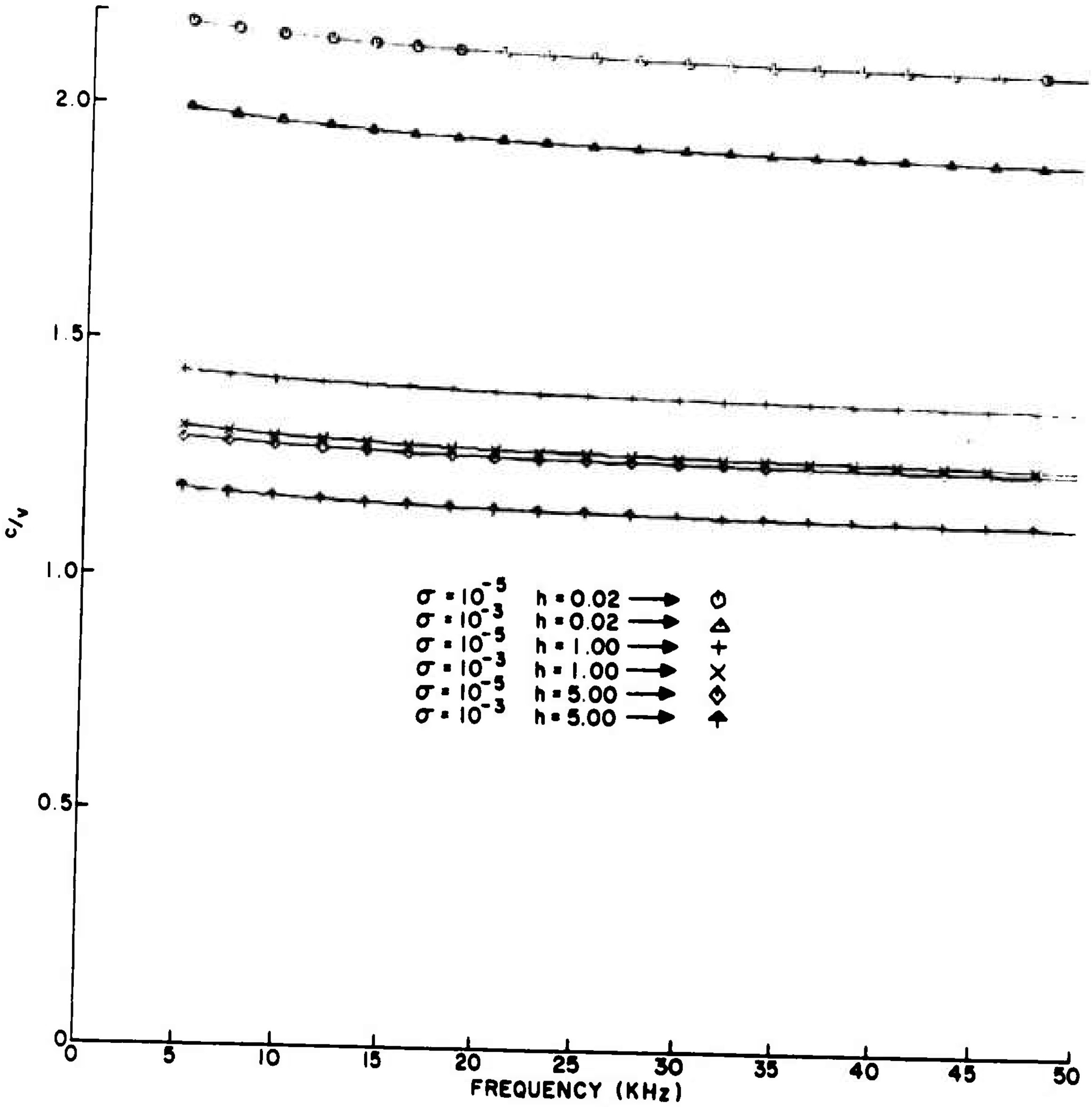


FIGURE 10. Wave Velocity Along Conductor Near Earth
(No. 6 wire radius = 0.2 cm)

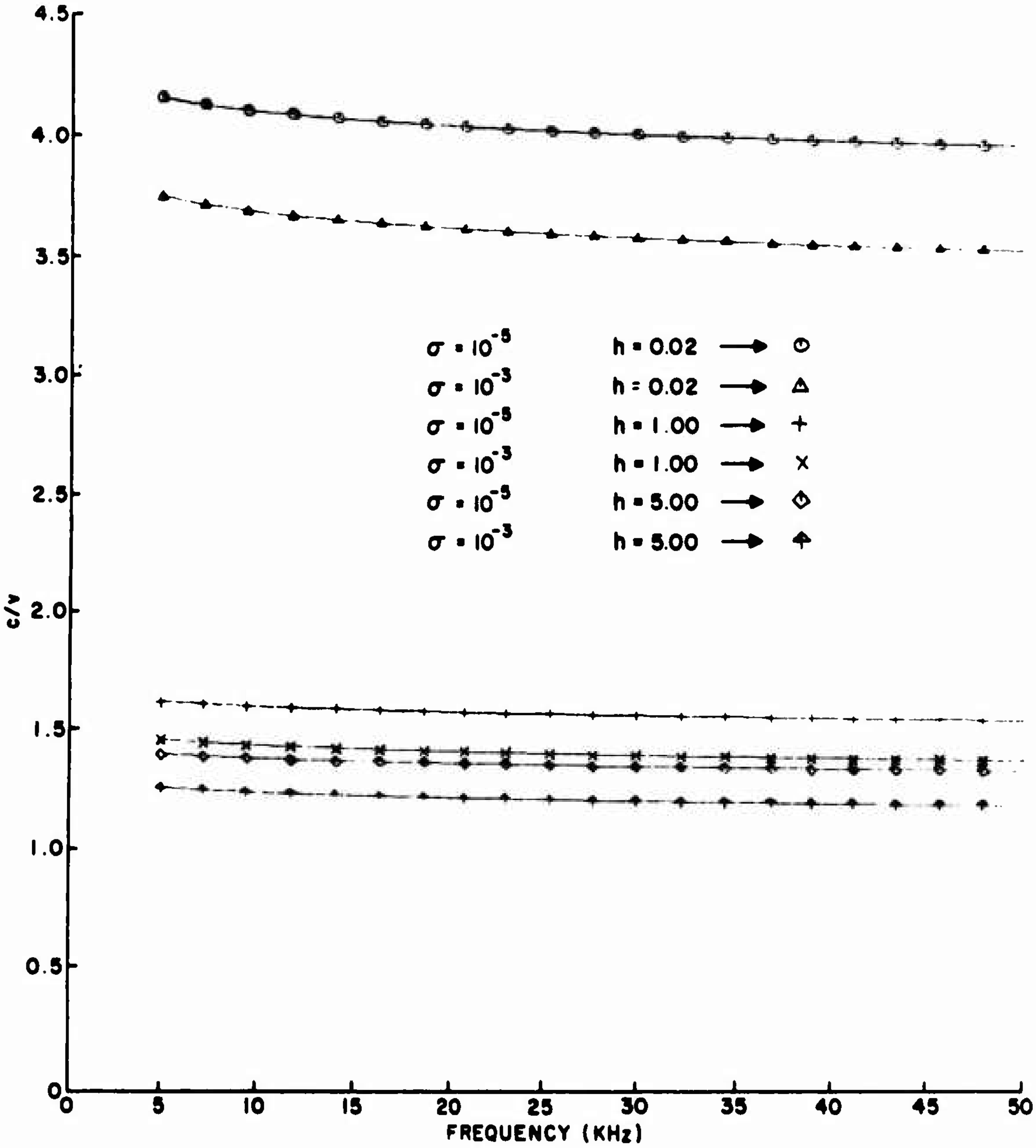


FIGURE 11. Wave Velocity Along Conductor Near Earth
(Wire radius = 2 cm)

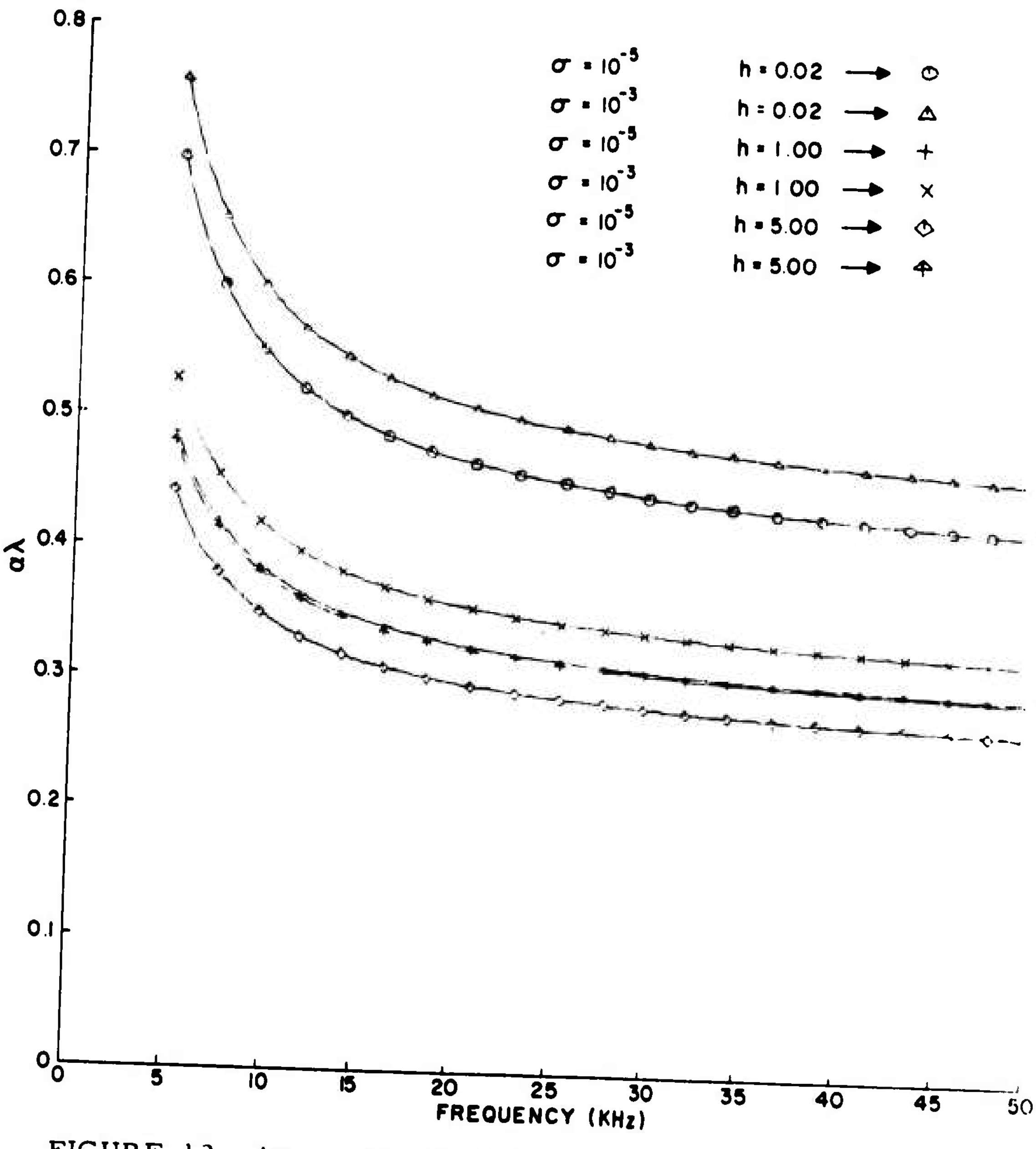


FIGURE 12. Attenuation Along Conductor Near Earth (No. 12 wire radius = 0.1 cm)

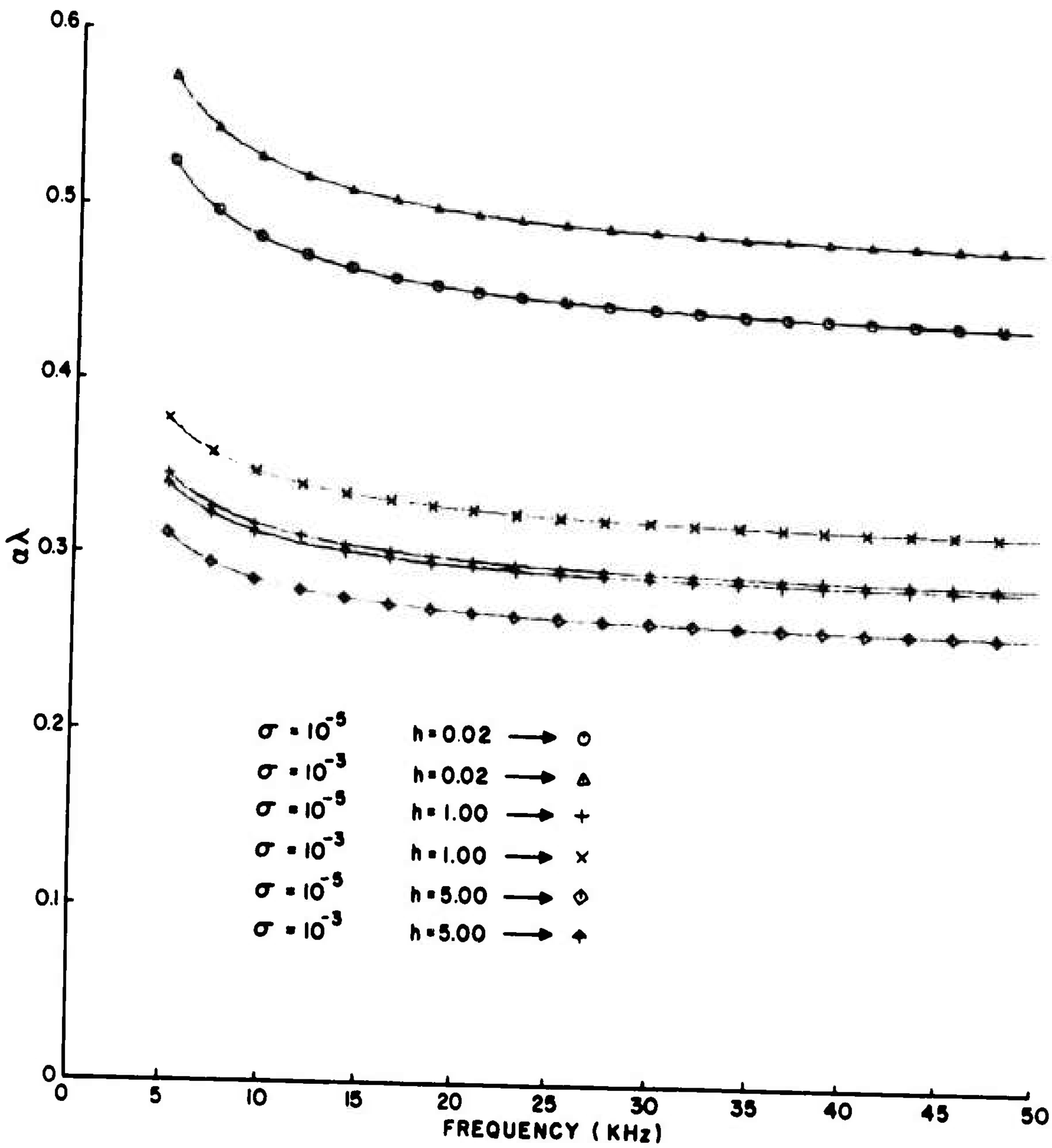


FIGURE 13. Attenuation Along Conductor Near Earth (No. 6 wire radius = 0.2 cm)

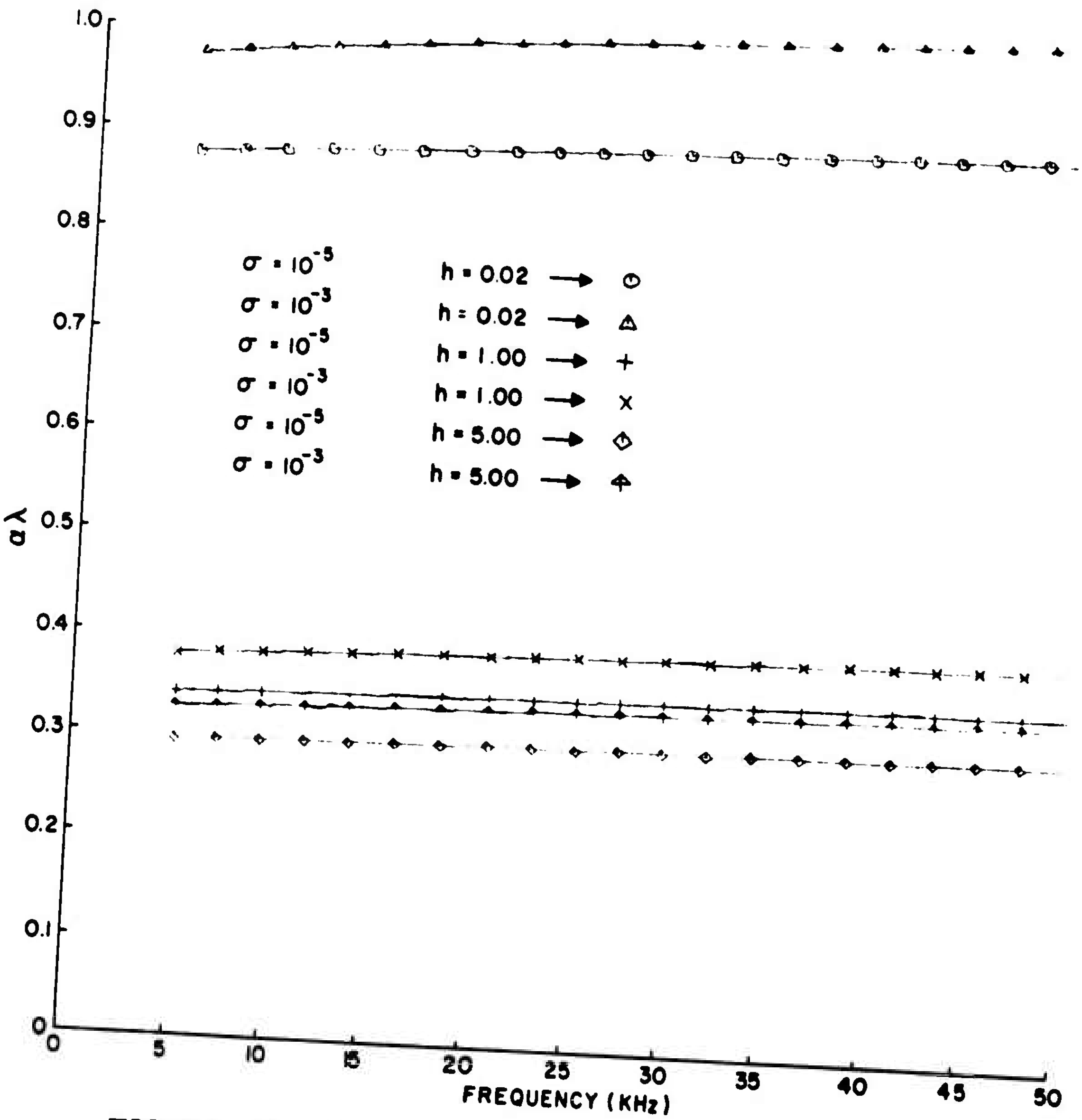


FIGURE 14. Attenuation Along Conductor Near Earth
(Wire radius = 2 cm)

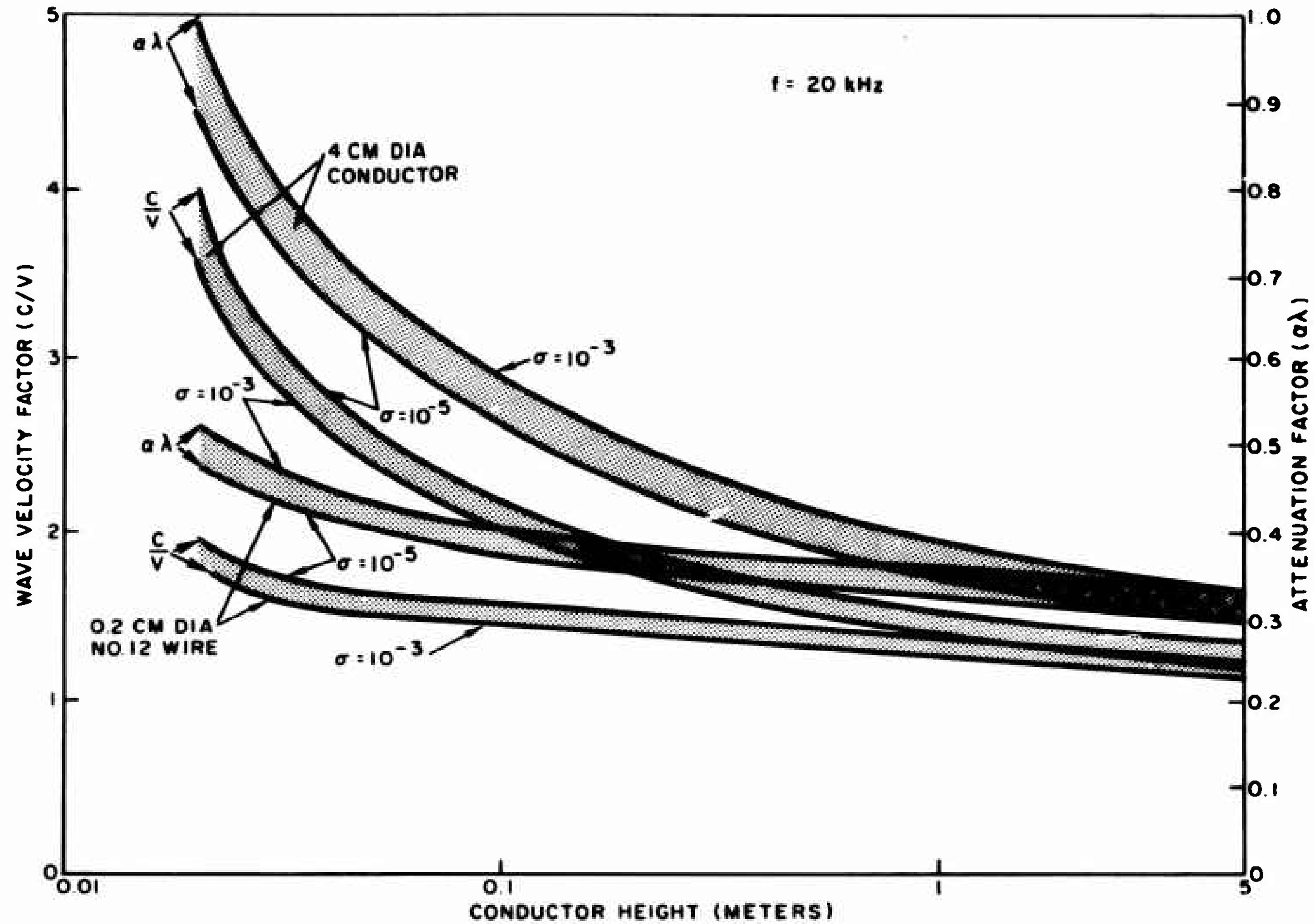


FIGURE 15. Wave Velocity and Attenuation of a Conductor Near Earth

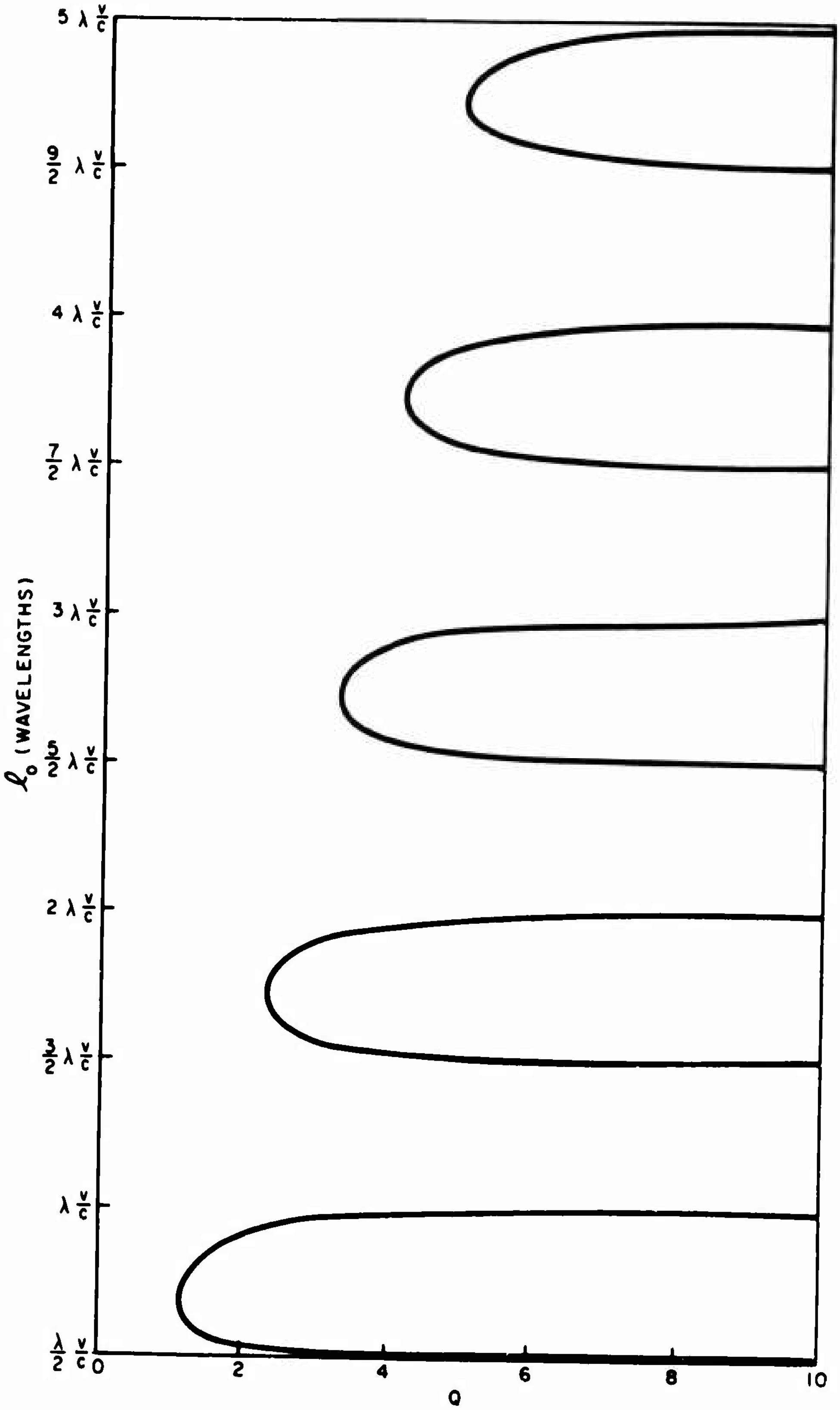


FIGURE 16. Lossy Dipole Resonant Lengths

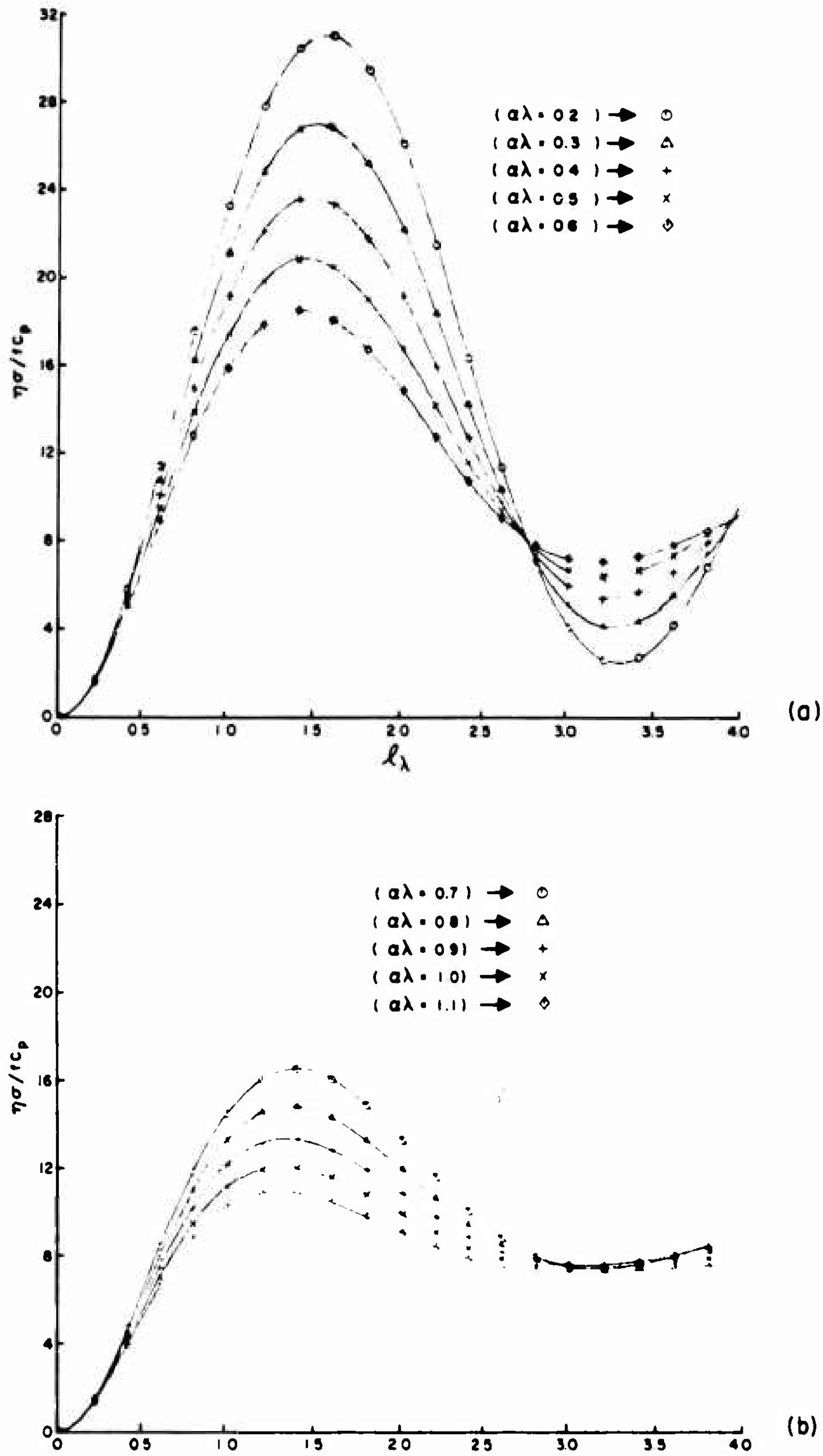


FIGURE 17. Efficiency of a Z_0-Terminated End-Fed Conductor Near Earth (c/v = 0.7)

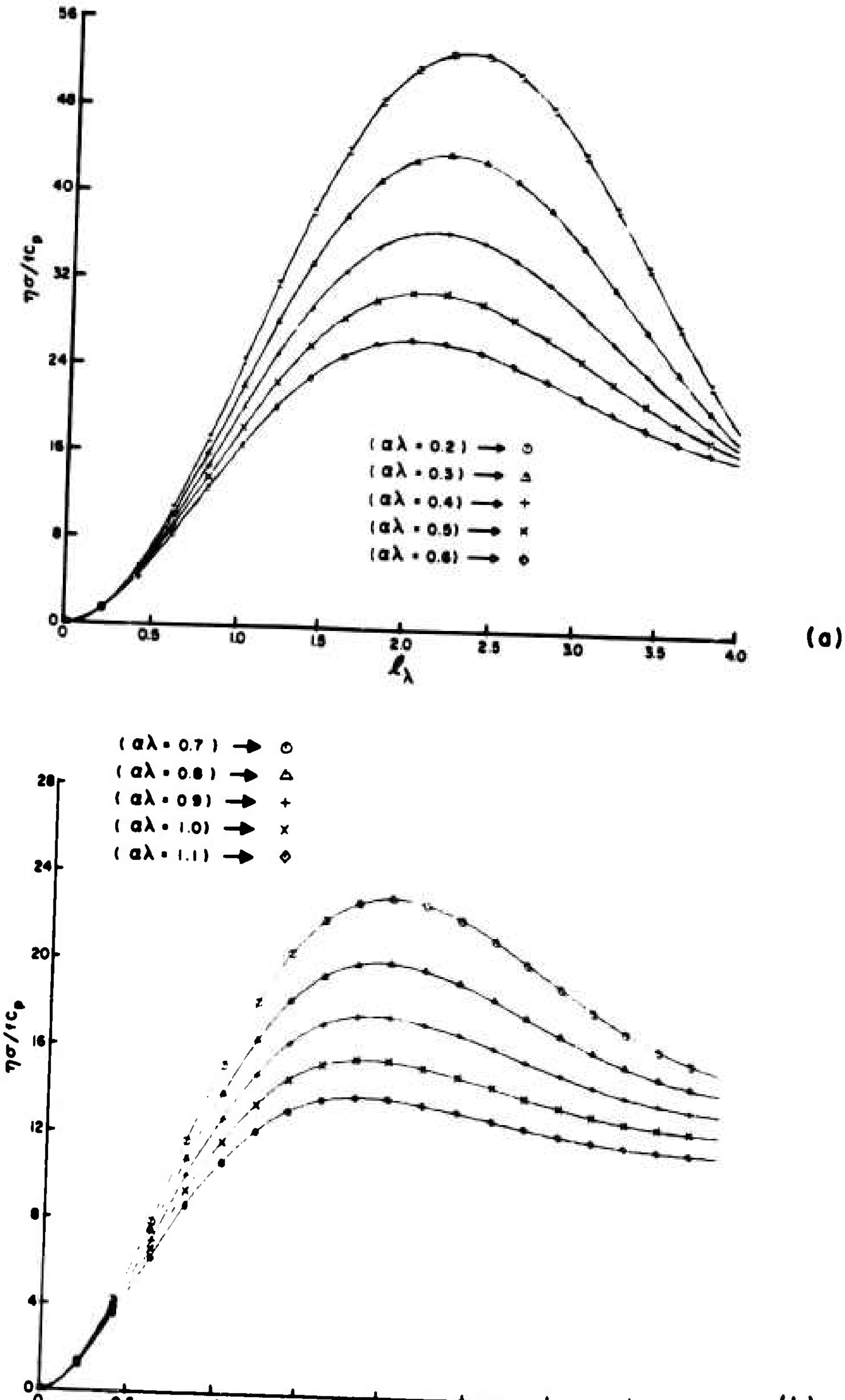


FIGURE 18. Efficiency of a Z_0-Terminated End-Fed Conductor Near Earth ($c/v = 0.8$)

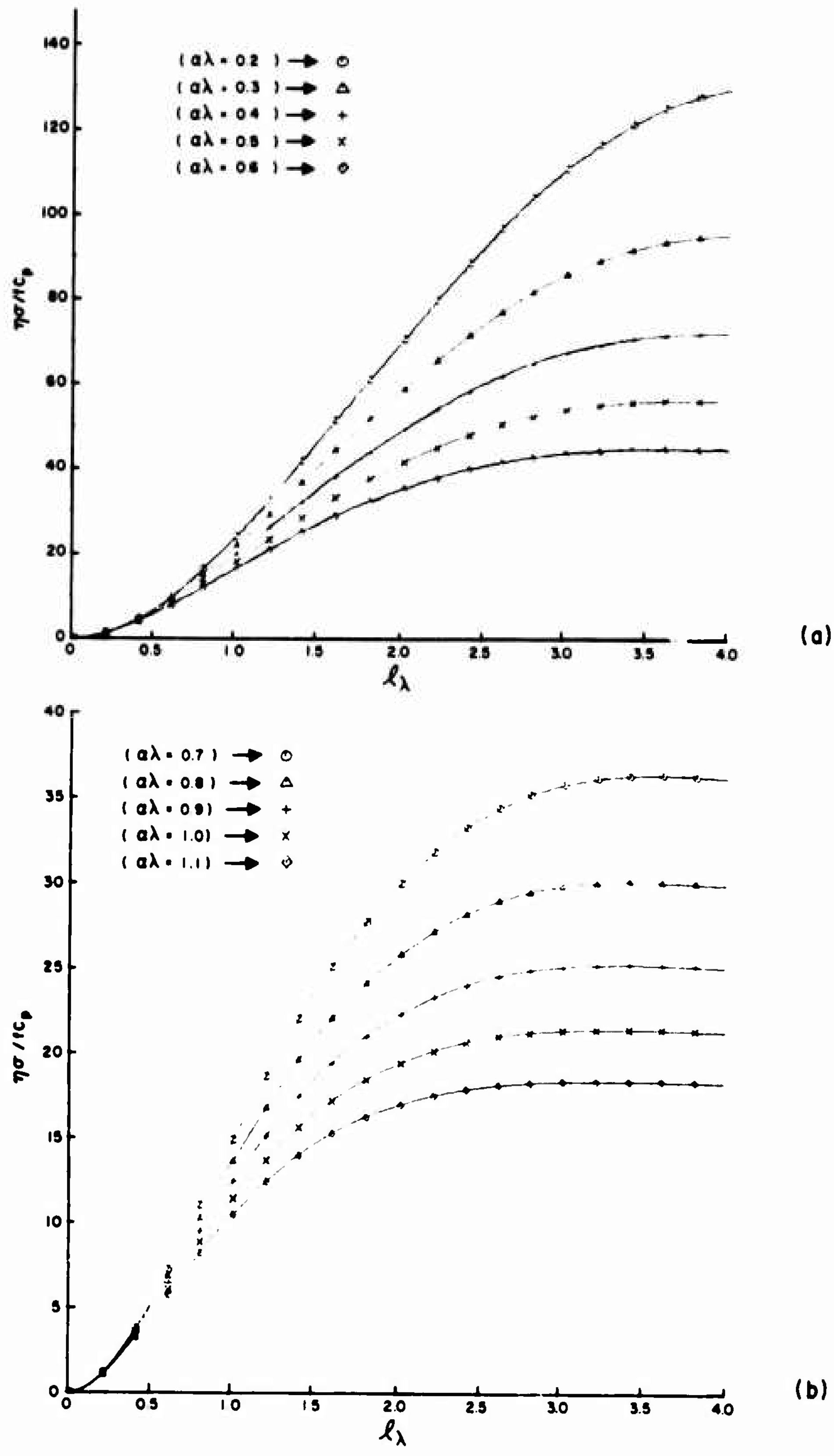


FIGURE 19. Efficiency of a Z_o-Terminated End-Fed Conductor Near Earth (c/v = 0.9)

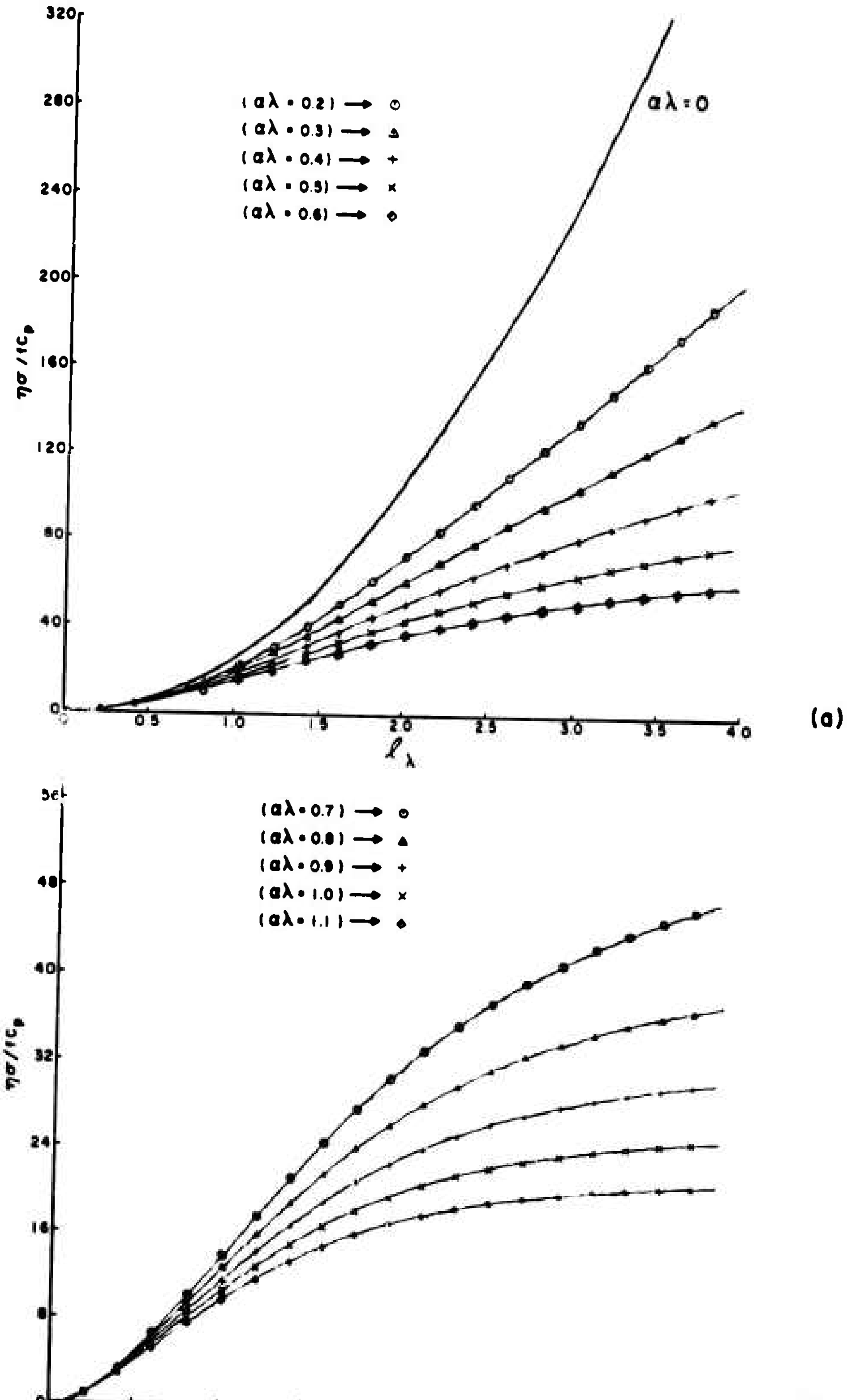


FIGURE 20. Efficiency of a Z_0-Terminated End-Fed Conductor Near Earth (c/v = 1.0)

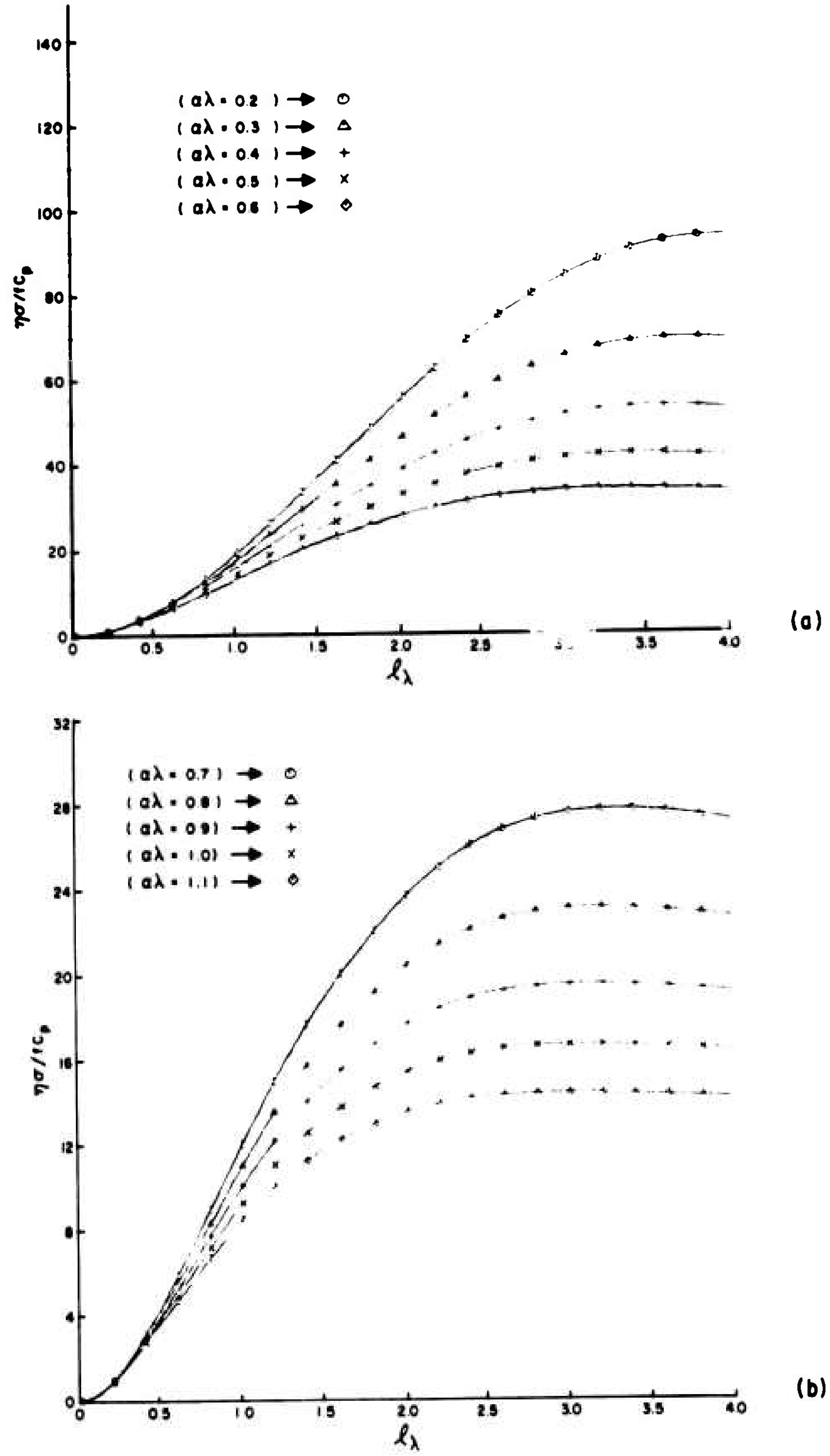


FIGURE 21. Efficiency of a Z_o-Terminated End-Fed Conductor Near Earth ($c/v = 1.11$)

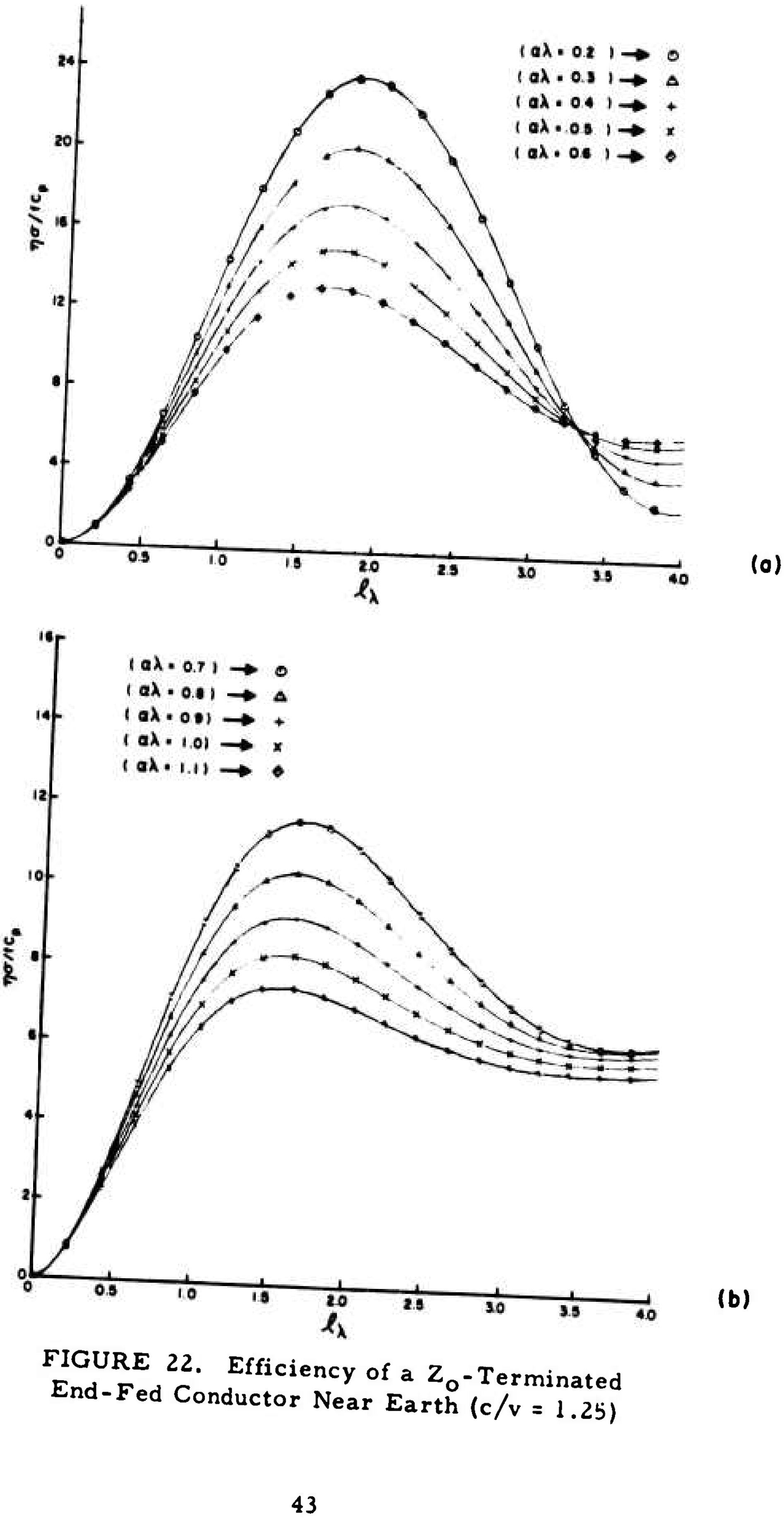


FIGURE 22. Efficiency of a Z_o-Terminated End-Fed Conductor Near Earth (c/v = 1.25)

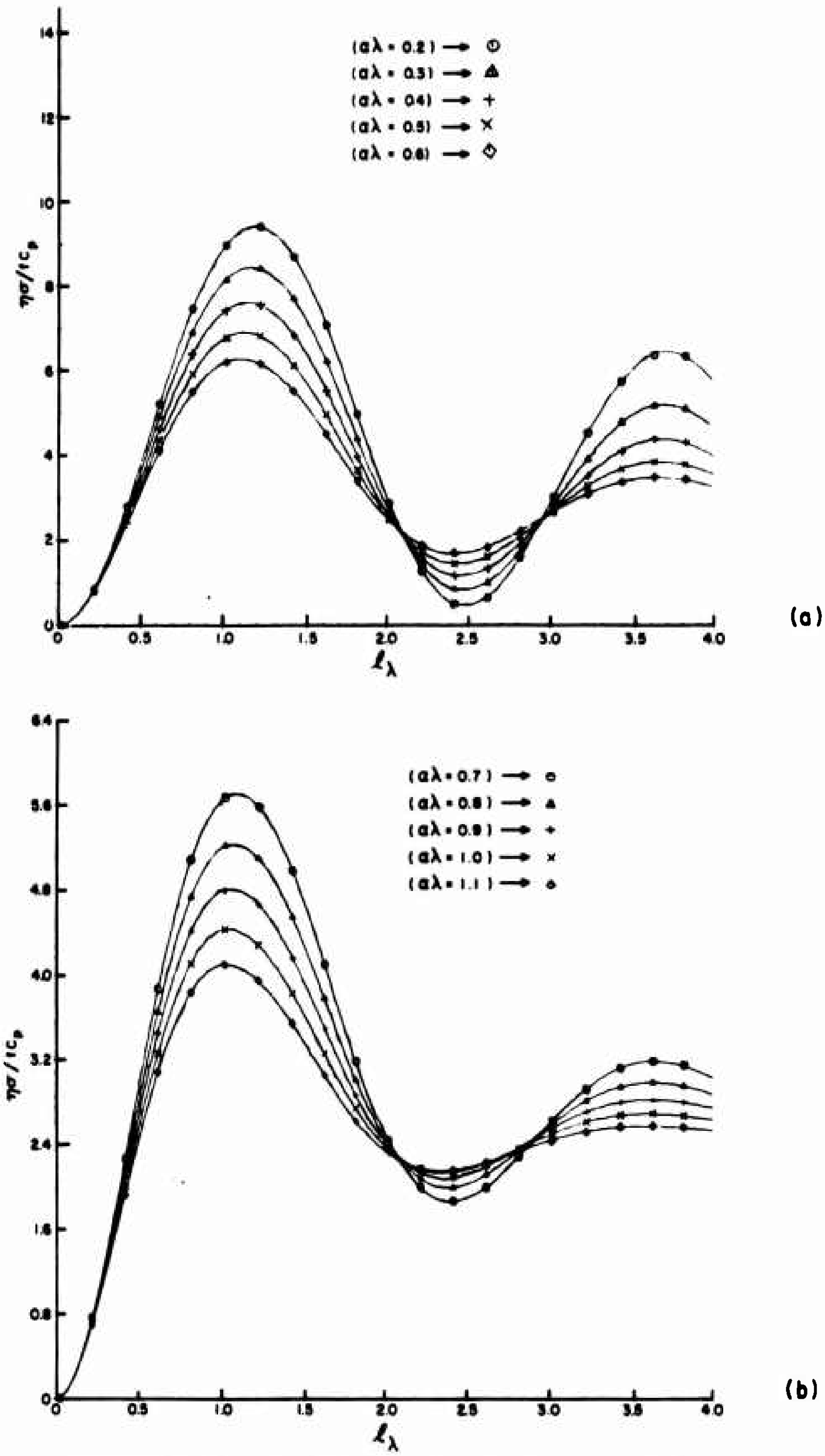


FIGURE 23. Efficiency of a Z_o-Terminated End-Fed Conductor Near Earth (c/v = 1.4)

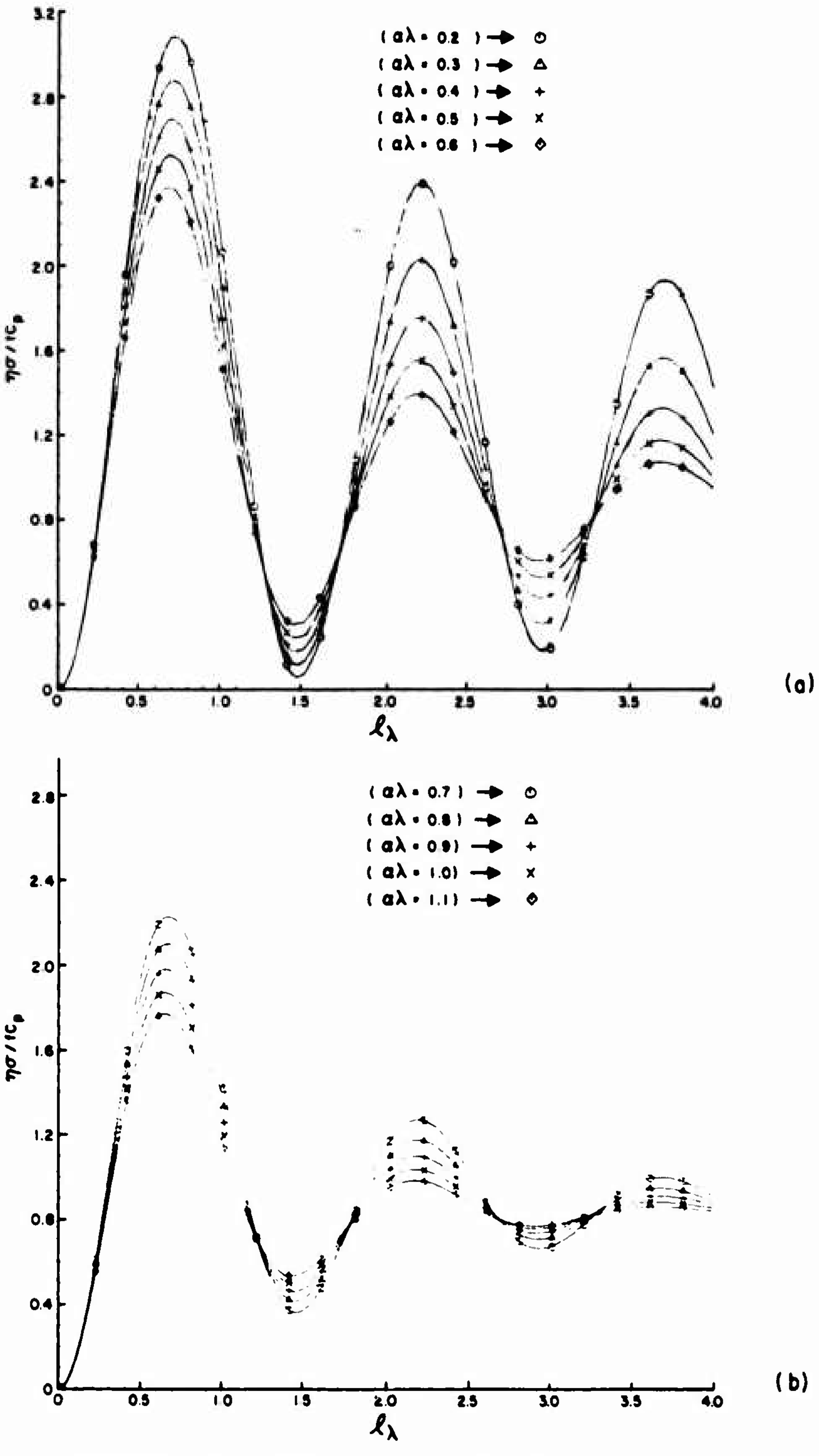
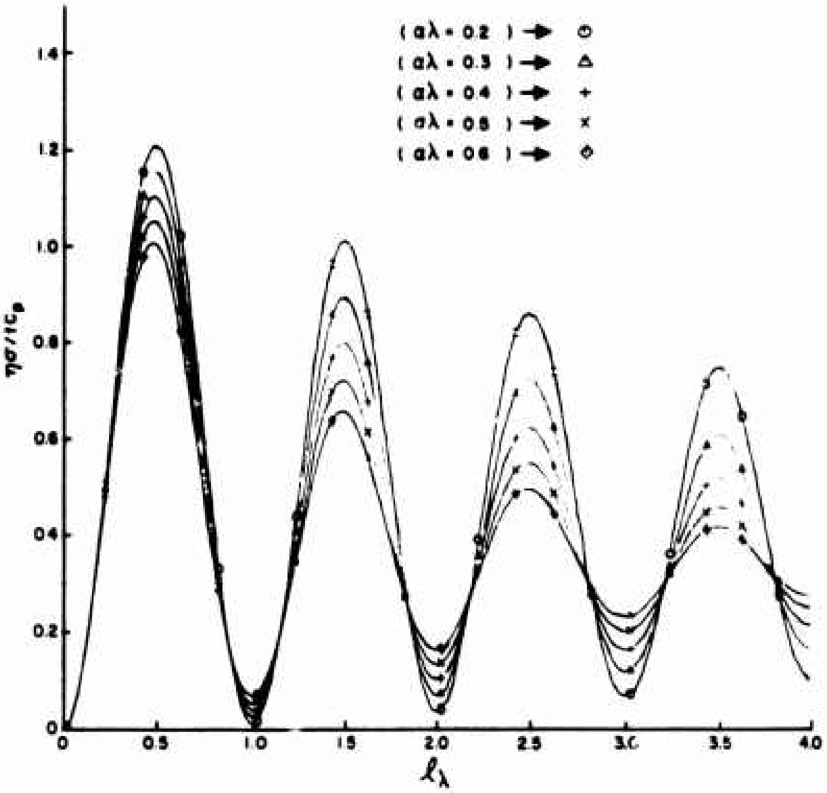


FIGURE 24. Efficiency of a Z_0-Terminated End-Fed Conductor Near Earth (c/v = 1.67)



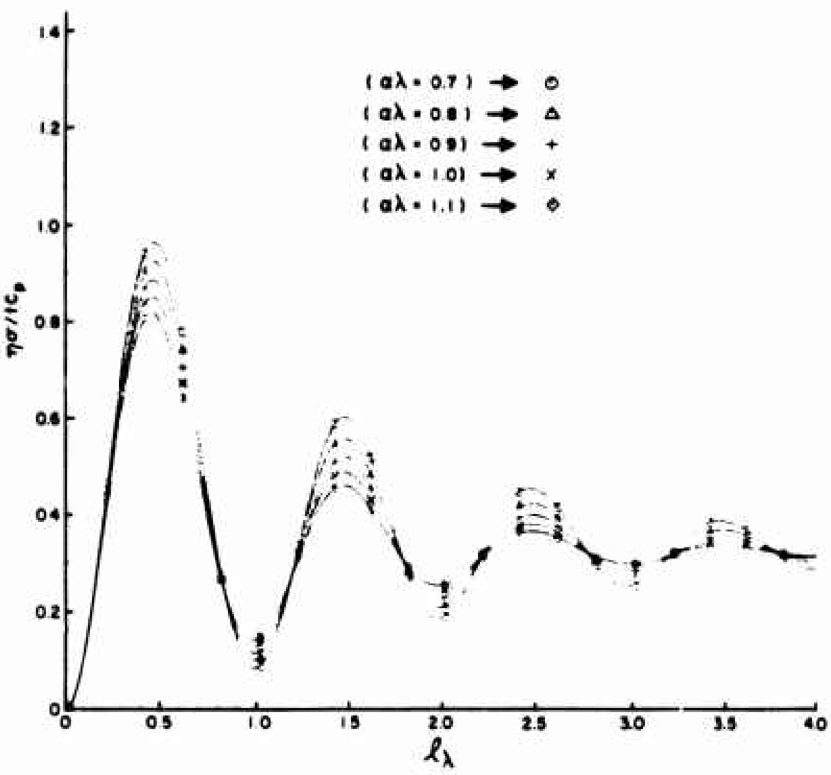


FIGURE 25. Efficiency of a Z_0-Terminated End-Fed Conductor Near Earth ($c/v = 2.0$)

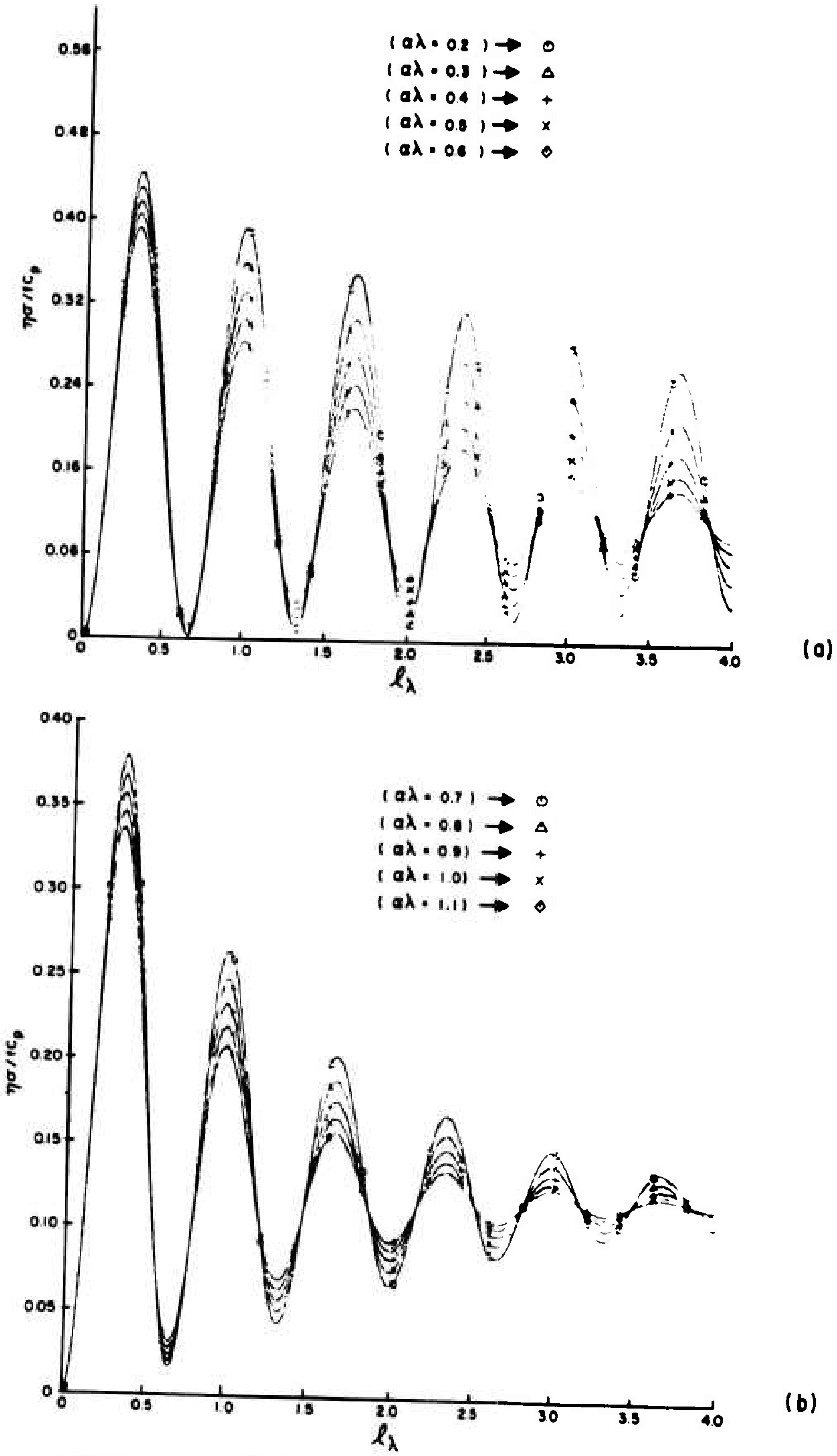


FIGURE 26. Efficiency of a Z_0-Terminated End-Fed Conductor Near Earth ($c/v = 2.5$)

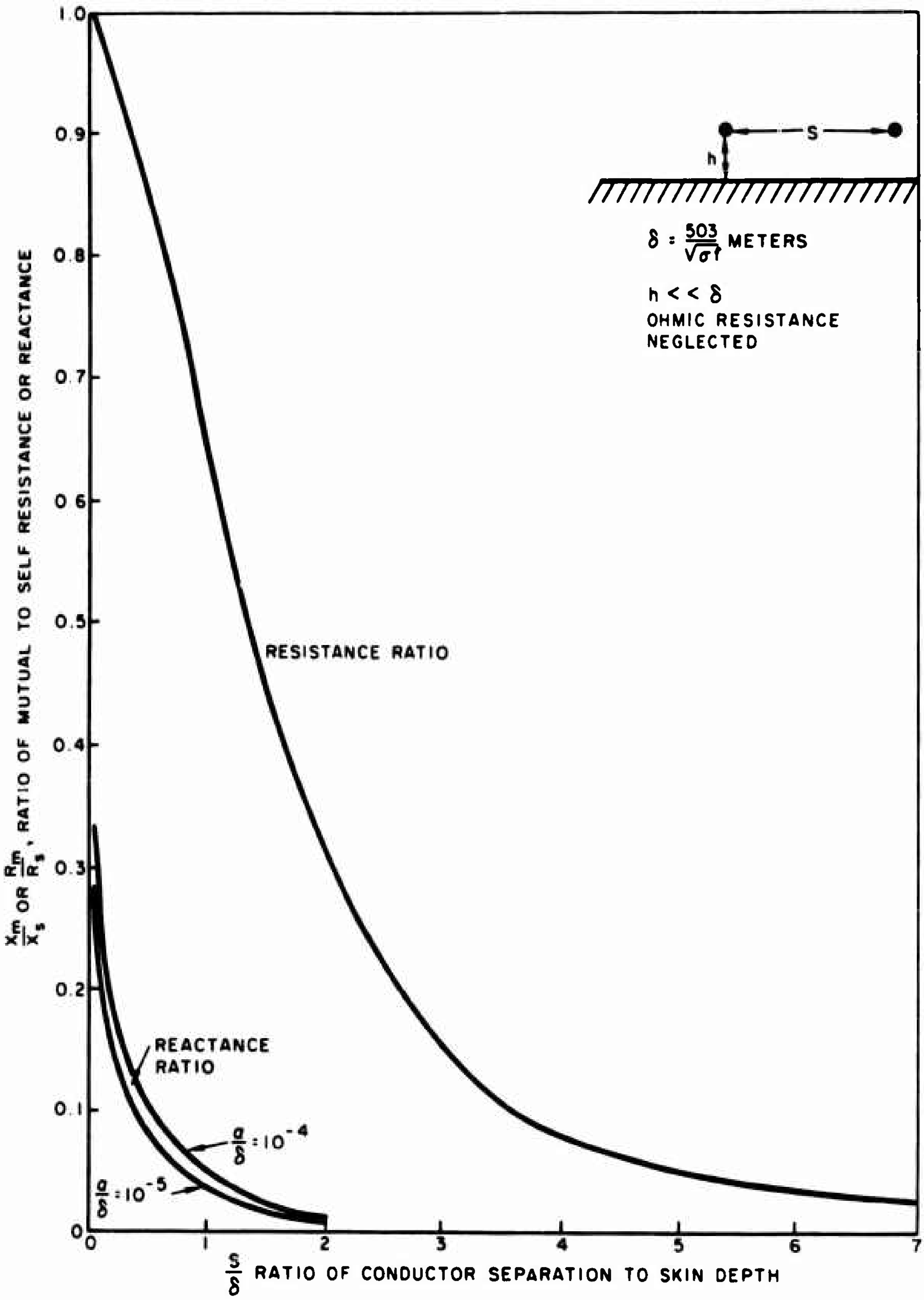


FIGURE 27. Ratio of Mutual Impedance to Self Impedance for Two Parallel Conductors Near Earth

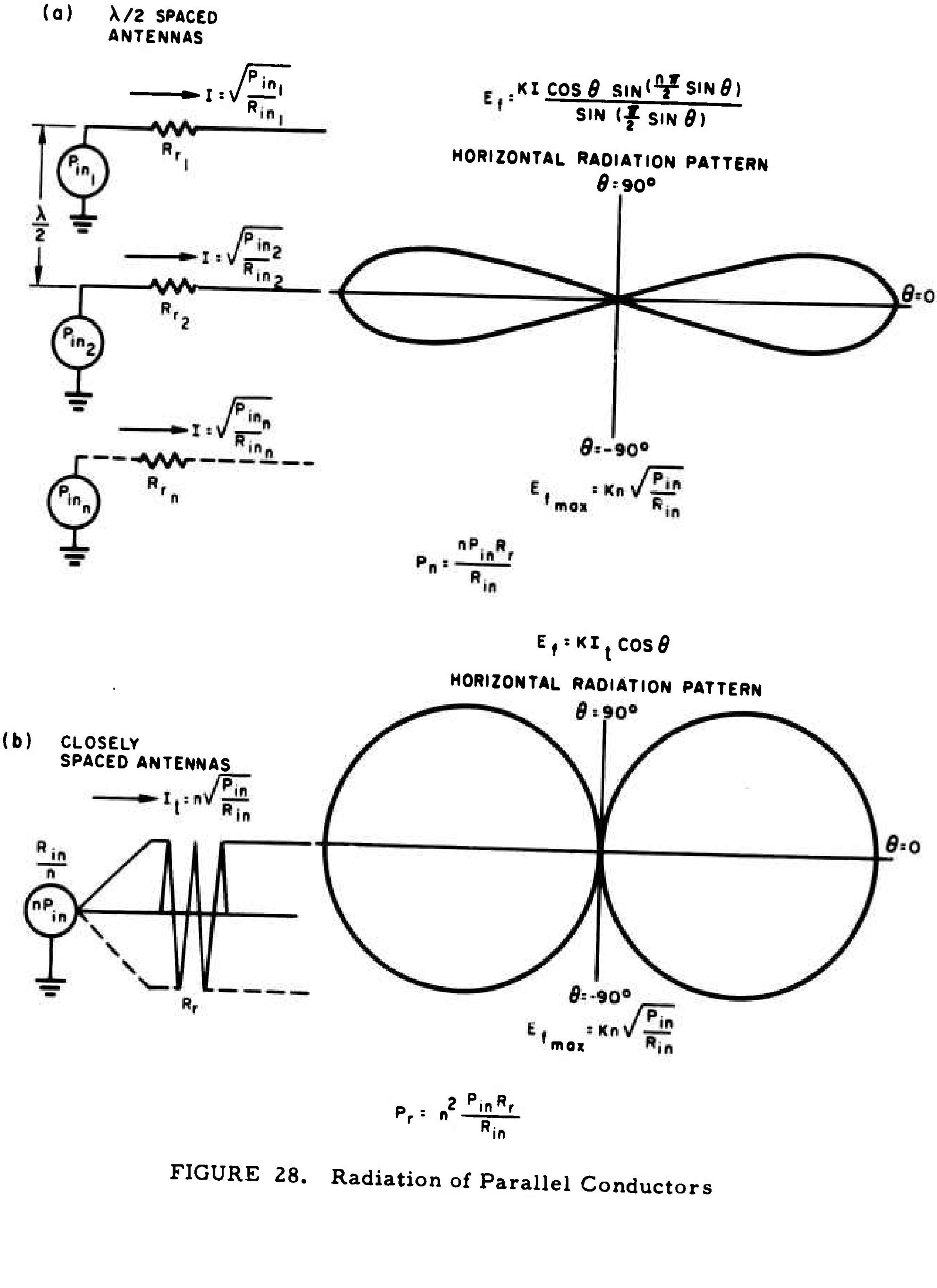


FIGURE 28. Radiation of Parallel Conductors

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13. ABSTRACT

Problems encountered in VLF radiation are outlined and variations of the horizontal dipole are proposed as a solution. Equations are derived for the radiation patterns, input impedance, and efficiency of the broad-band antenna (terminated in its characteristic impedance) and of the narrow-band antenna (open-terminated) where the antenna is fed at any position along its length. Data are presented to substantiate that (1) the horizontal dipole near the earth exhibits super gain when several parallel conductors are used; (2) the radiation efficiency of "n" closely spaced conductors is "n" times greater than when the same conductors are spaced one-half wavelength apart; and (3) the lossy horizontal dipole normally does not resonate when its length is a multiple of one-half wavelength. The "lossy lengthening factor" is shown in a plot of the resonant length as a function of "Q." This plot applies generally to all lossy dipoles.

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